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MASAO HIBINO

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CHAPTER 1. INTRODUCTION

1-1 PREFACE

In recent years, electrical communication technology has made a startling progress, not only in the domestic communication but also in the overseas communication. In many kinds of electrical communication systems, electrical filters have played extremely important roles. It is no exaggeration to say that the progress of filter technology has advanced that of communication systems. Recent communication systems require filters of low cost and small volume maintaining high performances. Needless to say, much efforts have been paid to this regard. However, in order to satisfy high performances, a lot of complex calculation process is needed. To solve this problem, the idea of computer aided design (CAD) has been introduced.

In this study, emphasis is placed upon the CAD methods for electrical filters. The historical review of filter design and the significance of this study will be described in the following sections.

1-2 HISTORICAL REVIEW OF FILTER DESIGN

Since the filter design theory by "image parameter method" was introduced by O.J.Zobel in 1923, many filter design theories based on this method have been investigated. However, this method had basically some difficulties to require many matching conditions. Furthermore, to realize a filter with sharp

cut off characteristics was very difficult. Overcoming these difficulties, S.Darlington, W.Cauer and H.Piloty have developed the design theory by "operational parameter method". The Chebyshev approximation theory for band pass filter based on this method has been developed by H.Watanabe, while the above mentioned people dealt with low pass filter. They all gave the transfer function with Chebyshev characteristic in a closed form. However, a given specification is, in many cases, arbitrary or does not always require Chebyshev characteristic. Therefore, a designed filter with Chebyshev characteristic may have an unreasonable characteristic for the given specification.

A.J.Gibbs has developed the iterative approximation technique by using the filter characteristic function $\psi(p)$ to meet an arbitrary specification. His method fixes cut off frequency and, therefore, after approximation, $\psi(p)$ converges with some constant extra insertion loss in the stop band.

In the author's method, on the contrary, the cut off frequency moves and, therefore, after approximation, $\psi(p)$ converges without any extra loss in both the passband and the stopband. This transfer function has a minimum group delay characteristic and a reasonable loss characteristic under a given order of filter.

Next, the development of realization technique by positive reactance elements will be reviewed. S. Darlington introduced the fact that no-pole filter, which has no attenuation pole in a finite frequency range, is realized by

positive reactance elements. The problem, "what is the realization condition of filter having attenuation poles in a finite frequency range?", was solved by T.Fujisawa for the low pass filter case. He also introduced the realization technique. For a band pass filter with unsymmetric transfer characteristic, H.Watanabe solved this problem.

For a band stop filter, this problem was solved by the author for the first time in which he gave a realization condition together with synthesis technique.

Recently, the digital signal processing (DSP) technique application to an electrical communication system, has been energetically investigated, because it is very attractive in regard to several aspects, such as cost, performance, size and reliability. In DSP technique, digital filters play essential roles. Progress in semiconductor technology, applicable to LSI's, has given a big influence on electrical filter realization technique. In other words, the digital filter technique, the idea of which appeared more than twenty years ago, has been introduced to a real world by using LSI technology. The digital filter design technique is fundamentally based on the classical network theory. However, it must be noted here that the digital filters are expressed in a sampled system.

Furthermore, since the coefficients and variables in digital filters are expressed by finite word length, quantization error effects must be considered. In spite of many contributions in this regard, it can be said that a fully practical

method has never been developed. The author has developed the digital filter design program system which contains various new approaches to minimize the quantization error effects. This contribution will be a step for a more practical design method.

1-3 SIGNIFICANCE AND SUMMARY OF THIS STUDY

With the development of digital computer, many studies on computer aided design, called CAD, have been done. For designing filters which accomplish high performances, CAD technique must be adopted. In this case, it is necessary to pay attention in the following two points.

- (i) Easy access to digital computer.

- (ii) Less computation process for saving computing time.

Throughout this study, the strong attention has been paid to these points.

This paper consists of two parts;

- (1) Reactance filter design (Chapters 2-5).

- (2) Digital filter design (Chapters 6-7).

From another point of view, we can divide this paper into two contents of network approximation and realization theory. Chapters 2, 5, 6 and 7 deal with approximation theory, while Chapters 3, 4, 5 and 6 are concerned with realization theory.

In Chapter 2, a general approximation theory is investigated. The problem how to determine a reasonable cut off frequency and an optimum filter degree for an arbitrary

prescribed specification in the frequency domain, is solved by taking fully advantage of CAD technique.

In Chapter 3, an approximation theory introduced in Chapter 2 is applied to the design of a band stop filter with unsymmetric transfer characteristics and, furthermore, a practical realization theory for this type of band stop filter is introduced. By this work, an important problem on the filter realization theory remained for band stop filter, has been solved for the first time.

In Chapter 4, a fully automated design method of the Norton transformation, which is one of the processes of band pass ladder filter design and is to make the equivalent circuit transformation from the original element values into the practical element values, is developed by using a heuristic approach. This method makes possible to release the filter engineer from his hand work, who suffered from the complex Norton transformation process.

In Chapter 5, a design theory of a four-port directional filter to be used in a submarine cable system which requires extremely high reliability, is introduced. This design theory can reduce the number of filter elements with excellent performances. The directional filters designed by this theory are widely used for submarine cable systems in Japan.

In Chapter 6, a computer aided design program system for digital filter is presented. First, the general property of

digital filter and some problems related to an actual design are described and then, a new program system is explained. Finally, the digital filter used in TDM-FDM transmultiplexer is shown as an example, which is a typical application to digital signal processing.

In Chapter 7, a simultaneous approximation technique in both frequency and time domains for an infinite impulse response digital filter is introduced. The transfer function with equiripple stopband attenuation is expressed in a sampled system and, by using this transfer function, time response approximation is carried out. This method is applied to the design of the Nyquist roll off digital filter for use of data transmission system.

In Chapter 8, the results obtained throughout this study are summarized.

1-4 SYMBOLS APPEARED IN THE PAPER

Symbol	Meaning and definition
A_s	attenuation loss
A_e	return loss
Ω	angular frequency in a continuous system
P	complex variable in a continuous system
$S(p)$	operating transfer factor $S(p) = C \cdot H \frac{g(p)}{f(p)}$
$\psi(p)$	characteristic function $\psi(p) = H \frac{h(p)}{f(p)}$ $S(p) \cdot S(-p) = 1 + \psi(p) \cdot \psi(-p)$
$T(p)$	reflecting transfer factor
$H(p)$	transfer function $H(p) = \frac{1}{S(p)}$
N	degree for $g(p)$ or $h(p)$
M	degree for $f(p)$
n_0	degree of attenuation pole at zero frequency
n_∞	degree of attenuation pole at infinite frequency
q_i or Q_i	pole
Y	admittance matrix
S	scattering matrix
Π	unit matrix
k_i or h_i	residue

Symbol	Meaning and definition
k_0	residue at zero frequency
k_∞	residue at infinite frequency
R	resistance
L or ℓ	inductance
C	capacitance
ϕ_i	transformer ratio
Ge or Ev	even part of polynomial
U_n	odd part of polynomial
Re	real part or real axis
Im	imaginary part or imaginary axis
LPF	low pass filter
HPF	high pass filter
BPF	band pass filter
BSF	band stop filter
f_s	sampling frequency (Hz)
T	sampling rate (second) $T = \frac{1}{f_s}$
ΔT	clock rate (second)
ω	angular frequency in a sampled system (or a discrete system)
z	complex variable in a sampled system (or a discrete system)
$H(z)$	transfer function
FIR	finite impulse response
IIR	infinite impulse response
$x(n)$	n th number in the input sequence
$y(n)$	n th number in the output sequence

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1-5-1 Publication

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Japanese)" National Convention Record, 1987, IECE of Japan, Mar. 1980

- (29) M.Morimoto, Y.Kobayashi and M.Hibino: "An Optimal SAW Filter Design using FIR Design Technique (in Japanese)" Autumn Convention Record, The Acoustical Society of Japan (to be published)

* sign indicates the papers directly related to this study.

CHAPTER 2. GENERAL APPROXIMATION THEORY

2-1 INTRODUCTION

A loss specification given to a filter is, in general, arbitrary as shown in Fig. 2-1, in which a loss characteristic of filter should be outside of the shaded area.

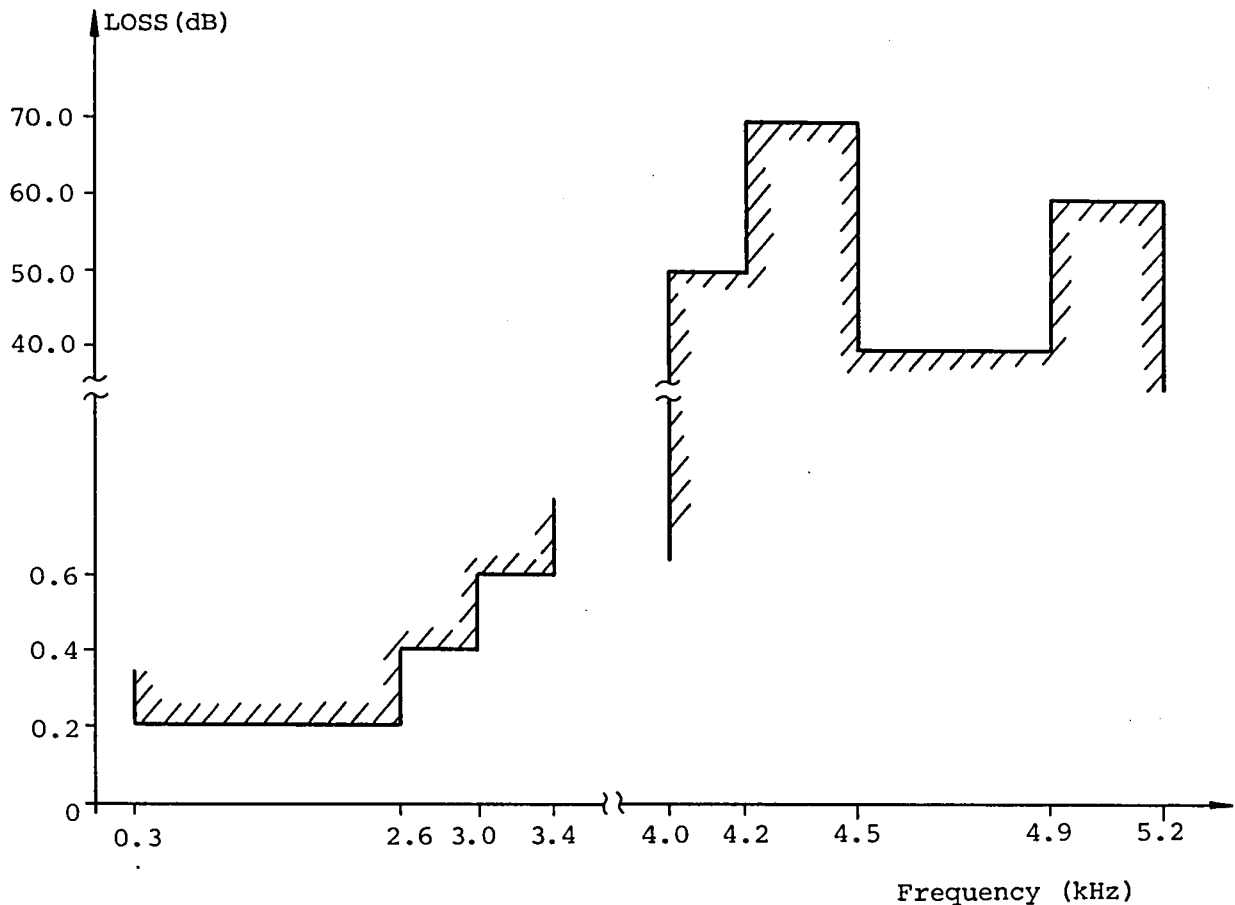


Fig. 2-1 An example of loss specification.

For example, assuming that the filter shown in Fig. 2-1 is used in a telephone transmission system, a stairway shape loss specification reflects the fact that the voice signal limited within 4.0 kHz frequency band has a frequency spectrum of a

convex form (called psophometer weighting). In other words, in this case, we do not need to have a flat passband and equiripple stopband filter.

A filter approximation theory with an arbitrary stopband and a Chebyshev passband characteristics has been developed by H.Watanabe⁽¹⁾, T.Fujisawa⁽²⁾, G.C. Temes⁽³⁾ and others. However, as clearly shown in the passband specification in Fig. 2-1, it is not necessary for a filter to have a Chebyshev passband ripple. This suggests a possibility to reduce a filter degree.

A.J.Gibbs⁽⁴⁾ has given an iterative approximation method to solve this problem, in which his filter has a weighted Chebyshev passband.

However, there still remained a big problem because they all, including Gibbs method, fixed the given cut off frequency. Therefore, after approximation, the transfer function has an extra attenuation loss. This causes a problem because an extra attenuation loss produces an increase of group delay $\tau(\Omega)$. It is well known that the loss deviation $D(\Omega)$ caused by Q characteristics of inductor and capacitor used in the network, is related to $\tau(\Omega)$ in the following relation⁽⁵⁾.

$$D(\Omega) \propto \tau(\Omega) \cdot \frac{1}{Q} .$$

Therefore, an increase of $\tau(\Omega)$ produces an increase of $D(\Omega)$.

The author has developed an improved approximation method, in which a reasonable cut off frequency is determined so that

the transfer function exactly meets the specifications with reasonable tolerance and without extra attenuation loss(6). Examples of the conventional method and the author's new method are compared in the final section in this chapter.

2-2 FUNDAMENTAL PRINCIPLE

On a reactance transfer function, the following relations are well known.

$$A_S = A_e + 10 \log_{10} \psi(p) \cdot \psi(-p) \quad , \quad P = j\Omega \quad , \quad \Omega: \begin{array}{l} \text{angular} \\ \text{frequency} \end{array}$$

where

$$A_S = 10 \log_{10} S(p) \cdot S(-p)$$

$$A_e = 10 \log_{10} T(p) \cdot T(-p)$$

$S(p)$: operating transfer factor

$T(p)$: reflection coefficient factor

$\psi(p)$: filter characteristic function,

and, A_S and A_e express the attenuation loss and the return loss, respectively.

Since $A_S \gg A_e$ in the stopband, we have

$$A_S \approx 10 \log_{10} \psi(p) \cdot \psi(-p) . \quad (2-1)$$

Since $A_e \gg A_S$ in the passband, we have

$$A_e \approx -10 \log_{10} \psi(p) \cdot \psi(-p) . \quad (2-2)$$

Therefore, we can characterize the both bands by $\psi(p)$. Fig. 2-2 shows graphically the relations between A_S , A_e and $\psi(p)$. The relation between A_S or A_e and the error caused by the approximation of Eqs.(2-1) or (2-2), is shown in Fig. 2-3.

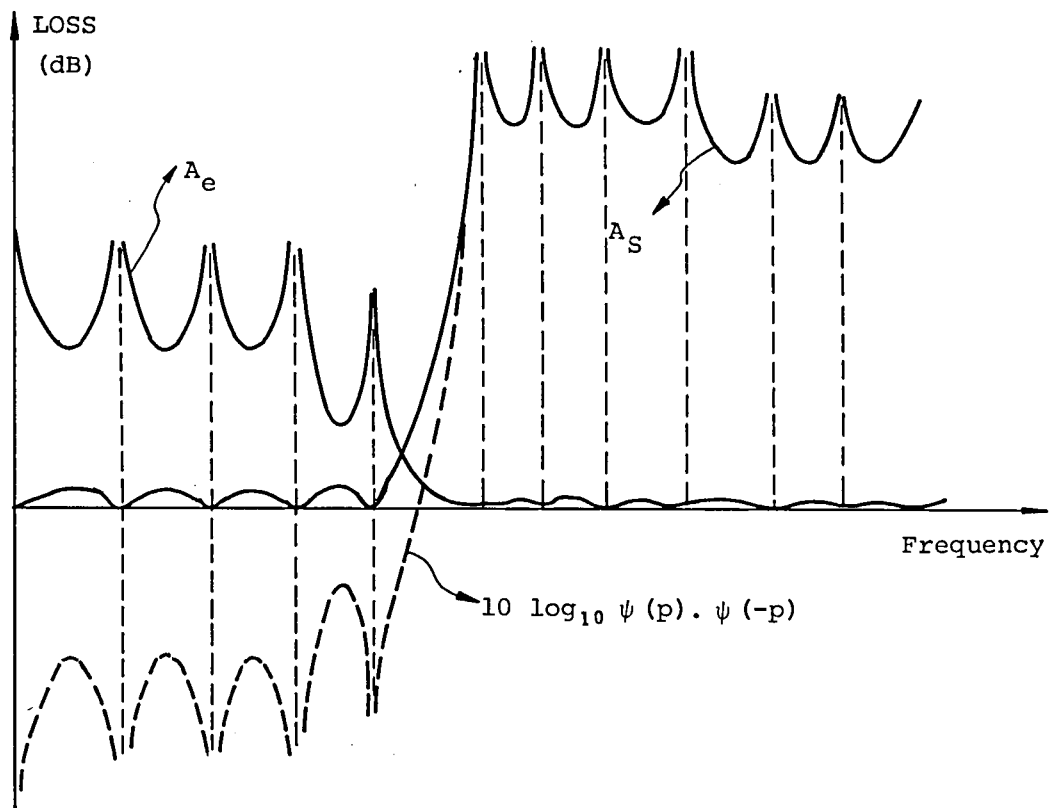


Fig. 2-2 The relation between A_e , A_s and $\psi(p)$.

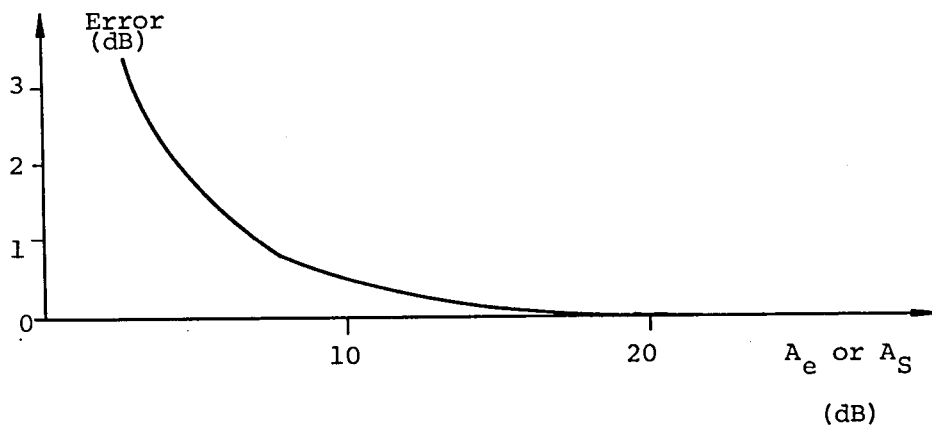


Fig. 2-3 The relation between A_e or A_s and the error caused by the approximation of Eqs. (2-1) or (2-2).

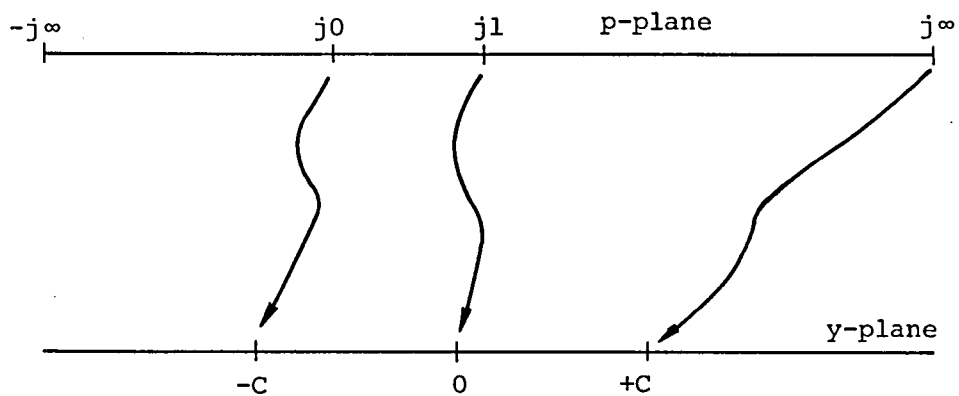
In case that A_e or A_s is larger than 10 dB which is general in actual design stage, the error is less than 0.5 dB.

2-3 FREQUENCY TRANSFORMATION

Since the frequencies to be considered here range from zero ($p=j0$) to infinity ($p=j\infty$), the imaginary axis in the p -plane must be transformed into the finite region by the following equation.

$$y = C \frac{p^2 + 1}{p^2 - 1} \quad (2-3)$$

where C is positive. By this linear frequency transformation, y is limited to the region from $-C$ to $+C$. This relationship is shown in the following figure.



$\psi(p)$ is, in general, given as follows(5),

$$\psi(p) = H \frac{h(p)}{f(p)} = H \frac{\prod_{\mu=1}^k (p^2 + c_{\mu}^2)^{\gamma_{\mu}} \cdot (p+c_0)^{\gamma_n}}{\prod_{v=1}^h (p^2 + Q_v^2)^{\gamma_v} \cdot p^{\gamma_0}}$$

where H is a constant and $h(p)$, $f(p)$ are polynomials of p .

The zeros and poles except for term $(p + C_0)^{\gamma_n}$ in $\psi(p)$ exist on the imaginary axis. When $h(p)$ is of odd degree, $\gamma_n = 1$, and when $h(p)$ is of even degree, $\gamma_n = 0$. γ_0 expresses the degrees of zero at $p=j0$ in $f(p)$.

Here, we define N , M , n_{∞} and n_0 as

$$N = 2(\gamma_1 + \gamma_2 + \dots + \gamma_k) + \gamma_n$$

$$M = 2(\gamma_1 + \gamma_2 + \dots + \gamma_h) + \gamma_0$$

$$n_{\infty} = N - M$$

$$n_0 = \gamma_0$$

n_{∞} and n_0 express the degrees of attenuation poles at infinite frequency and zero frequency, respectively.

In the low pass filter, we have the relations of $C_0 = 0$, $n_0 = 0$ and $N > M$.

In the band pass filter, we have the relations of $n_0 \neq 0$ and $N > M$.

$\psi(p) \cdot \psi(-p)$ can be transformed into $\Phi(y)$ in the y -plane through the substitution of Eq.(2-3).

$$\Phi(y) = H'^2 \frac{\prod_{\mu=1}^k (y + y_{1\mu})^{2\gamma_{\mu}} \cdot (y - c'_0)^{\gamma_n}}{\prod_{v=1}^h (y + y_{2v})^{2\gamma_v} \cdot (y - c)^{\gamma'_0}}$$

where

$$y_{1\mu} = c \frac{1 - c_{\mu}^2}{1 + c_{\mu}^2}$$

$$y_{2v} = c \frac{1 - q_v^2}{1 + q_v^2}$$

$$c'_0 = c \frac{c_0^2 + 1}{c_0^2 - 1}$$

$$H'^2 = H^2 \frac{\prod_{\mu=1}^k (1 + c_{\mu}^2)^{2\gamma_{\mu}} \cdot (c_0^2 - 1)^{2\gamma_n}}{\prod_{v=1}^h (1 + q_v^2)^{2\gamma_v}}$$

and

$$\gamma'_0 = 2 \sum_{\mu=1}^k \gamma_{\mu} + \gamma_n - (2 \sum_{v=1}^h \gamma_v + \gamma_0) \quad .$$

Now we define $A(y)$ as follows.

$$A(y) = 10 \log_{10} \Phi(y)$$

$$= 10 \log_{10} e \{ K_0 + H_0 + \sum_{\mu=1}^k 2\gamma_{\mu} \cdot \log_e (y + y_{1\mu})$$

$$- \sum_{v=1}^h 2\gamma_v \cdot \log_e (y + y_{2v}) \} \quad (2-4)$$

where

$$K_0 = \log_e K$$

$$K = \frac{(y - C'_0)^{\gamma_n}}{(y - C)^{\gamma'_0}}$$

and $H_0 = \log_e H'^2$.

$A(y)$ expresses a loss characteristic in the y -plane.

2-4 DETERMINATION OF OPTIMUM CUT OFF FREQUENCY AND OPTIMUM FILTER DEGREE

The function $A(y)$ has $k+h+1$ independent variables of $y_{1\mu}$, $y_{2\nu}$ and H_0 which are determined so that $A(y)$ exactly meets the both specifications in the passband and the stopband except at the passband edge frequency. This is shown in Fig. 2-4 in the case of 7 degree low pass filter which has 7 independent variables.

In this figure, $A(y)$ exactly meets the specifications at 7 points denoted by circles. We can also see that the prescribed passband edge frequency y_c moves to y_c' as if the specified cut off frequency is changed.

This method can be applied to the band pass filter, as shown in Fig. 2-5. In this figure, the band pass filter which has 10 independent variables, exactly meets the specifications at 10 points except at two passband edge frequencies. In this case, two cut off frequencies y_{-p} and y_{+p} move to y_{-p}' and y_{+p}' respectively.

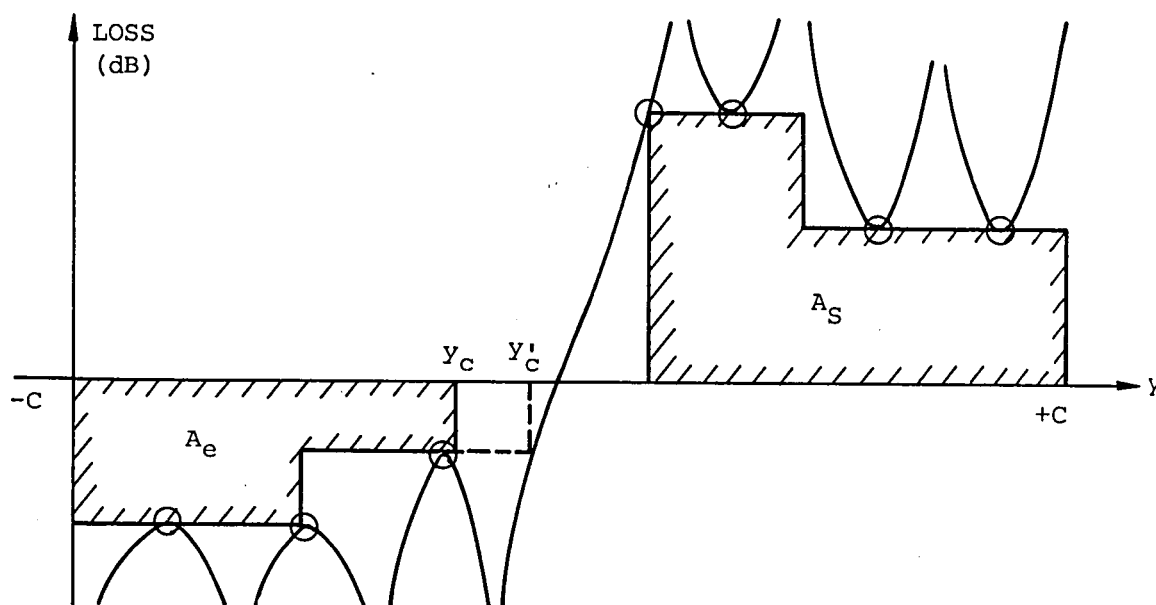


Fig. 2-4 Characteristic of $A(y)$ after approximation : low pass filter.

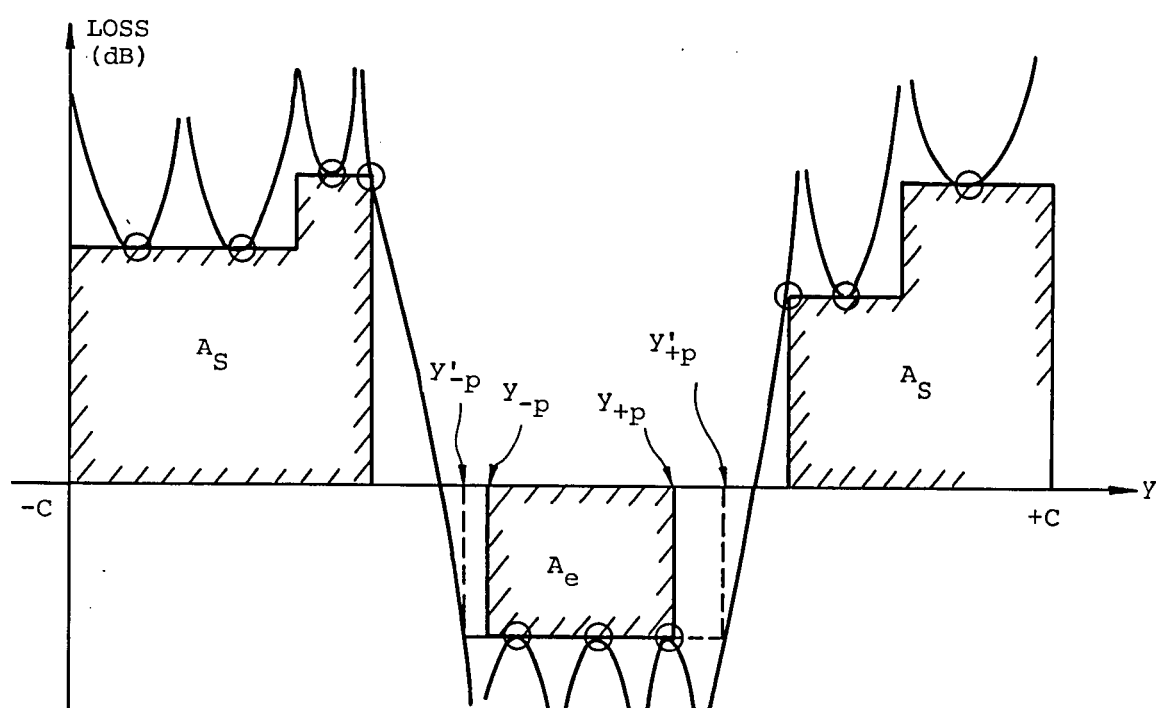


Fig. 2-5 Characteristic of $A(y)$ after approximation : band pass filter.

Without loss of generality, we can restrict the following discussions to the low pass filter case. After optimization, in the low pass filter, the following three cases arise, which are shown graphically in Fig. 2-6.

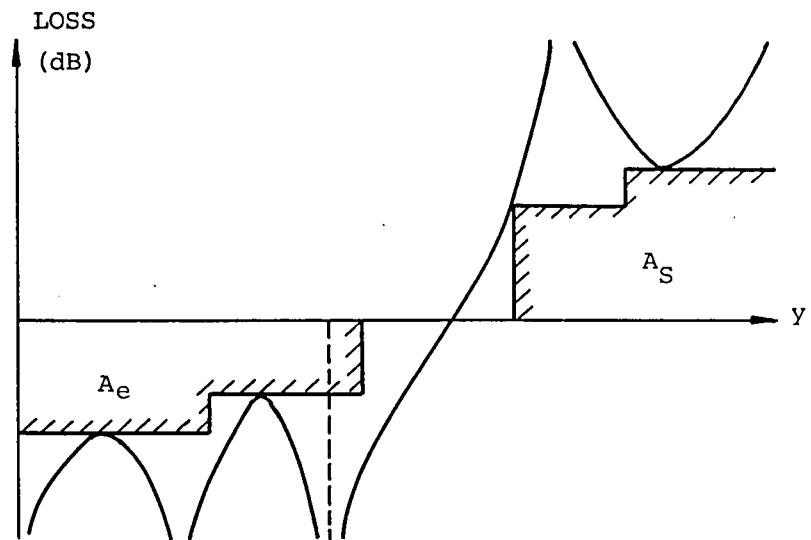
case 1: Specification is satisfied with reasonable tolerance which means that the zero nearest to the passband edge is within the specified passband (see Fig. 2-6-a). In this case, the initial degree of filter is reasonable to meet the specification.

case 2: Specification is unsatisfied (see Fig. 2-6-b). In this case, since the initial degree is under estimated, we should increase the degree, whose operation is carried out by increasing a pole and zero pair on the imaginary axis.

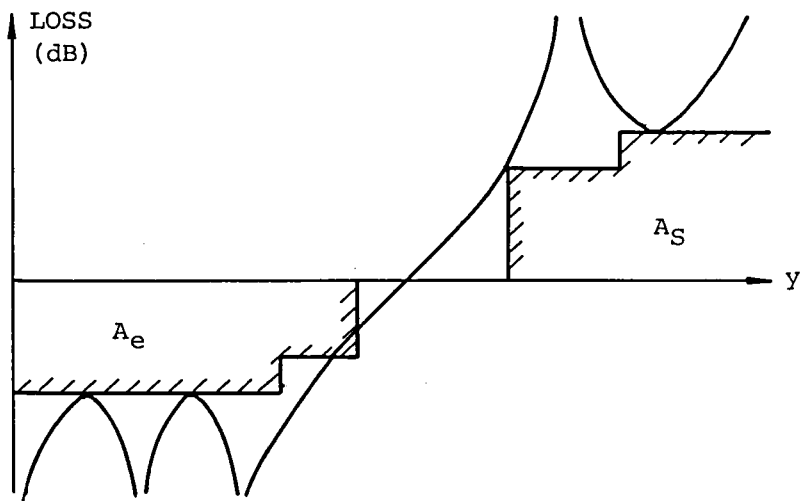
case 3: Specification is satisfied with too large tolerance which means here that the zero nearest to the passband edge is outside the given cut off frequency (see Fig. 2-6-c). In this case, since the initial degree is over estimated, we should decrease the degree, whose operation is carried out by eliminating a pole and zero pair on the imaginary axis.

In this procedure, n_0 and n_∞ are both fixed while N and M are changed. Through the above technique, we can determine the reasonable filter degree automatically by the judgement of computer.

(a) Case 1



(b) Case 2



(c) Case 3

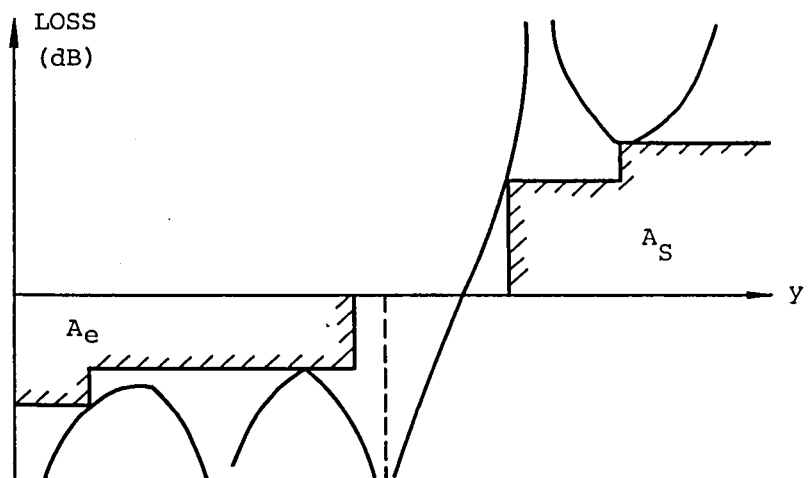


Fig. 2-6 Determination of an optimum filter degree.

2-5 ITERATIVE APPROXIMATION METHOD

Let us consider a low pass filter case. The frequency axis y is sectioned by pole and zero frequencies, they are

$$[-C, y_{11}] [y_{11}, y_{12}] [y_{12}, y_{13}] \text{ ---- } [y_{1k-1}, y_{1k}] \text{ and}$$

$$[y_s, y_{21}] [y_{21}, y_{22}] \text{ ---- } [y_{2h}, +C]$$

which consist of $k+h+1$ sections. y_s expresses the stopband edge frequency. The problem is to determine the $k+h+1$ parameters $y_{1\mu}$, $y_{2\nu}$ and H_0 in $A(y)$ so that the minimum difference $e_i(y_i)$ ($i=1,2, \text{ ---- } , k+h+1$) between the function $A(y)$ and the specification at $k+h+1$ points y_i becomes to zero, in which y_i indicates the frequency to give the minimum difference in i -th section. This is shown in Fig. 2-7.

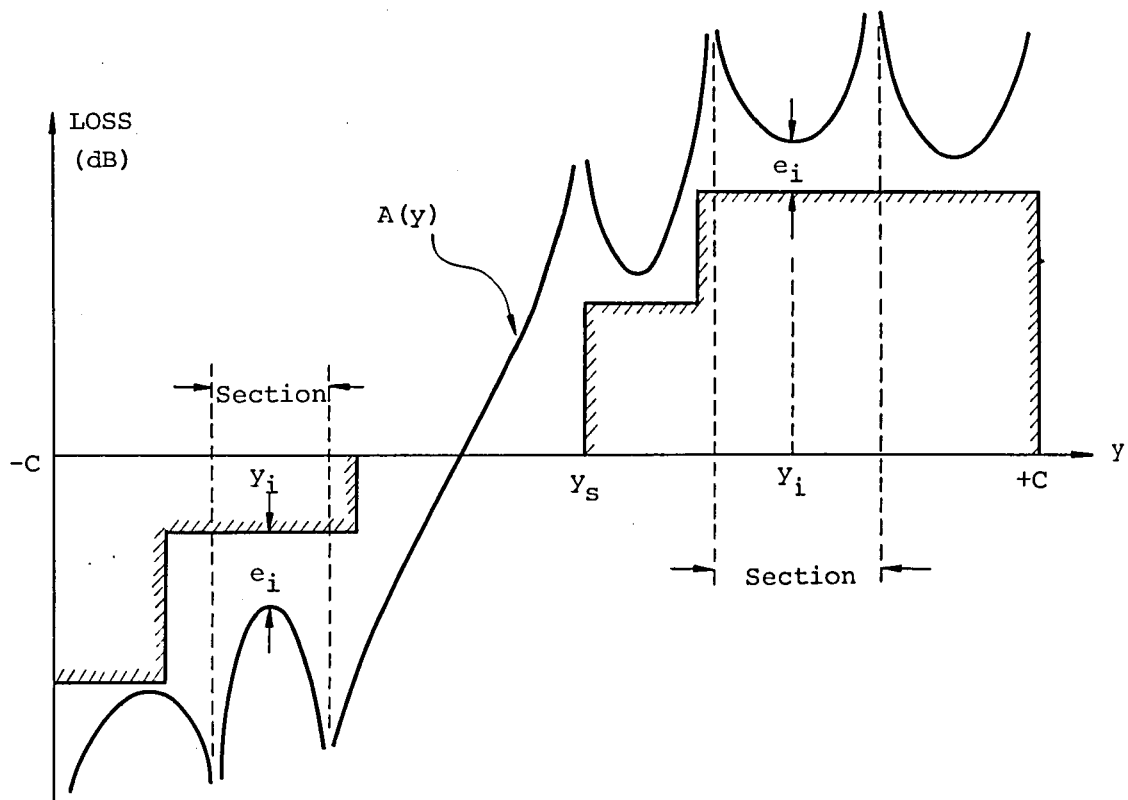


Fig. 2-7 Approximation in $A(y)$.

This problem can be solved by assuming a first order approximation with parameters $y_{1\mu}$, $y_{2\nu}$, and H_0 . By taking partial derivative of $A(y)$ in Eq.(2-4) with respect to $y_{1\mu}$, $y_{2\nu}$, and H_0 , we have a following equation.

$$10 \log_{10} e \left\{ \sum_{\mu=1}^k 2\gamma_{\mu} \frac{1}{y_i + y_{1\mu}} \Delta y_{1\mu} - \sum_{\nu=1}^h 2\gamma_{\nu} \frac{1}{y_i + y_{2\nu}} \Delta y_{2\nu} + \Delta H_0 \right\} = -e_i(y_i) \quad (2-5)$$

$i = 1, 2, \dots, k+h+1$

The parameters of a better approximation are

$$y_{1\mu}^* = y_{1\mu} + h_i \Delta y_{1\mu} \quad (2-6)$$

$$y_{2\nu}^* = y_{2\nu} + h_i \Delta y_{2\nu} \quad (2-7)$$

$$H_0^* = H_0 + \Delta H_0 \quad (2-8)$$

$$h_i = \min\{(y_{i+1} - y_{1\mu})/\Delta y_{1\mu}, 1\} \quad \text{for } \Delta y_{1\mu} > 0 \text{ (or } \Delta y_{2\nu})$$

$$h_i = \min\{(y_i - y_{1\mu})/\Delta y_{1\mu}, 1\} \quad \text{for } \Delta y_{1\mu} < 0 \text{ (or } \Delta y_{2\nu})$$

where * sign means new parameter. Here, h_i is introduced because new parameters are restricted not to exceed out of the range $[y_i, y_{i+1}]$. This comes from the reason of better convergence for iterations. This process is repeated until $e_i(y_i)$ ($i=1, 2, \dots, k+h+1$) are satisfied within an allowable approximation error ϵ_0 .

2-6 DESIGN FLOW CHART

For easy access to the program, it is desired to reduce the amount of input information given to the program. In this

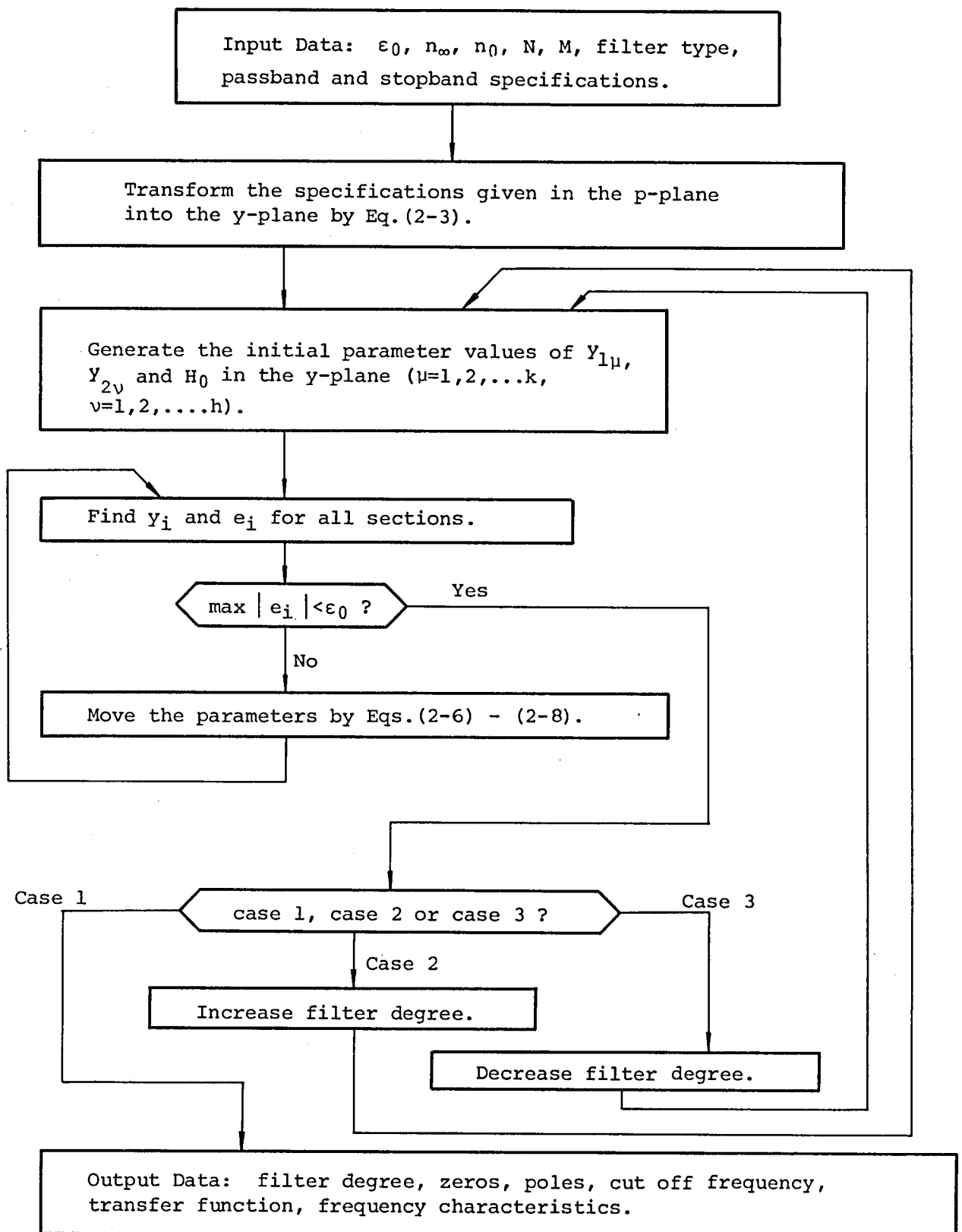


Fig. 2-8 Design flow chart.

program, the frequency characteristics specifications, ϵ_0 , n_∞ , n_0 , N , M and filter type of LPF, HPF, BPF, BSF are required. The filter degree N and M are varied in the course of the program. Initial parameter set is generated automatically in the program. Program flow chart is shown in Fig. 2-8.

2-7 DESIGN EXAMPLES

Four design examples are shown, where examples 1 - 3 are 9th degree low pass filter (referred to as LPF) and example 4 is 18th degree band pass filter (referred to as BPF). The shaded area in the following figures indicates the specifications for A_S and A_E . Example 1 is designed by the conventional Gibbs method. Since the given cut off frequency 1.84 kHz is fixed, this LPF has a 5 dB constant extra loss in the stop-band (see Fig. 2-9). Example 2 is designed by the new method. This LPF exactly meets the specification and the final cut off frequency is 1.87457 kHz (see Fig. 2-10). Fig. 2-11 shows the delay and loss characteristics in the passband with actual Q values for example 1 and 2, respectively. From Fig. 2-11, it is clearly shown that example 2 has better characteristics than example 1 in both loss and delay. Example 3 designed by the new method shows the case without equiripple passband in which the cut off frequency indicates 1.92518 kHz (see Fig. 2-12). Example 4 shows a typical BPF designed by the new method (see Fig. 2-13).

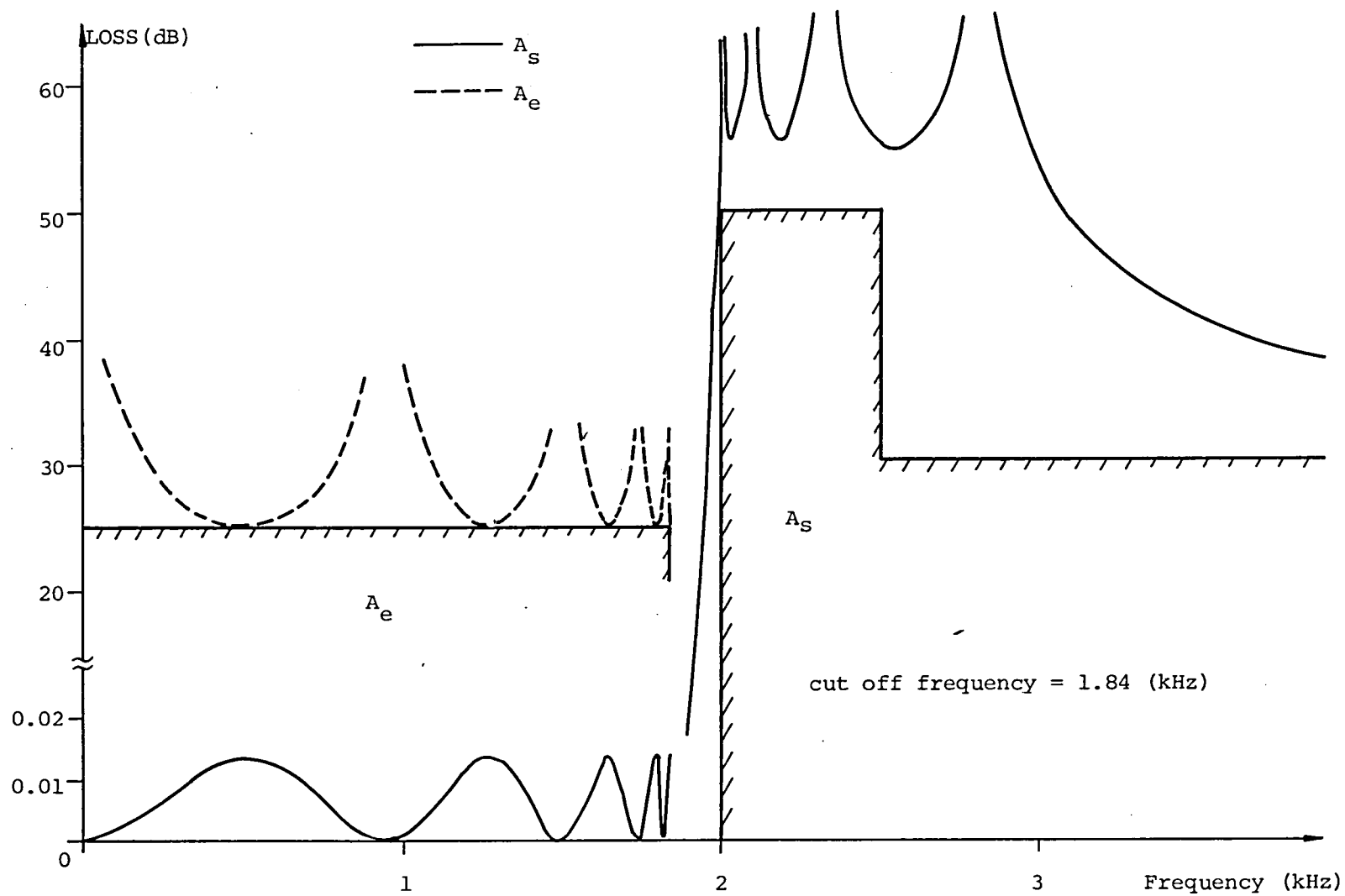


Fig. 2-9 Example 1 by the conventional method.

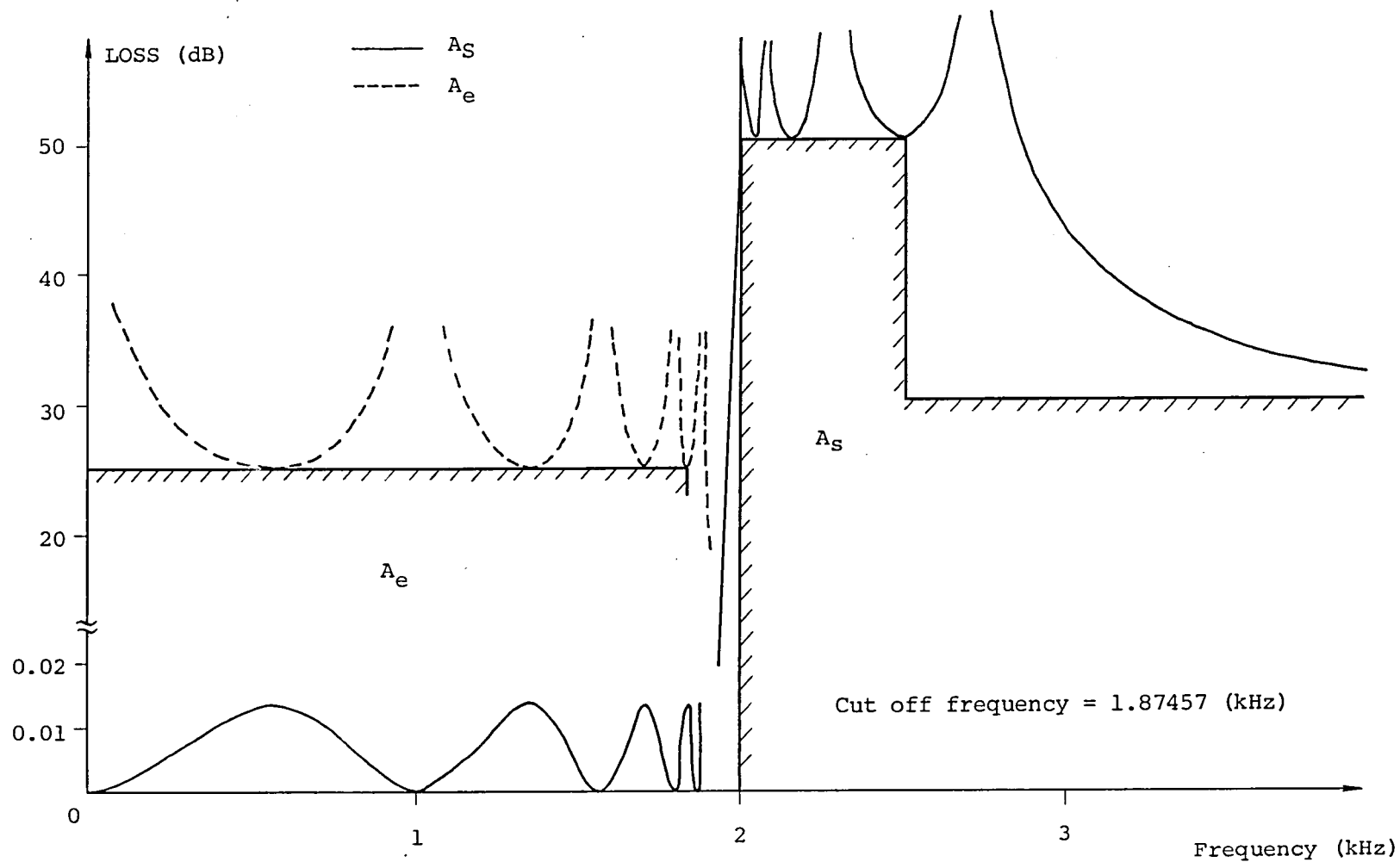


Fig. 2-10 Example 2 by the new method.

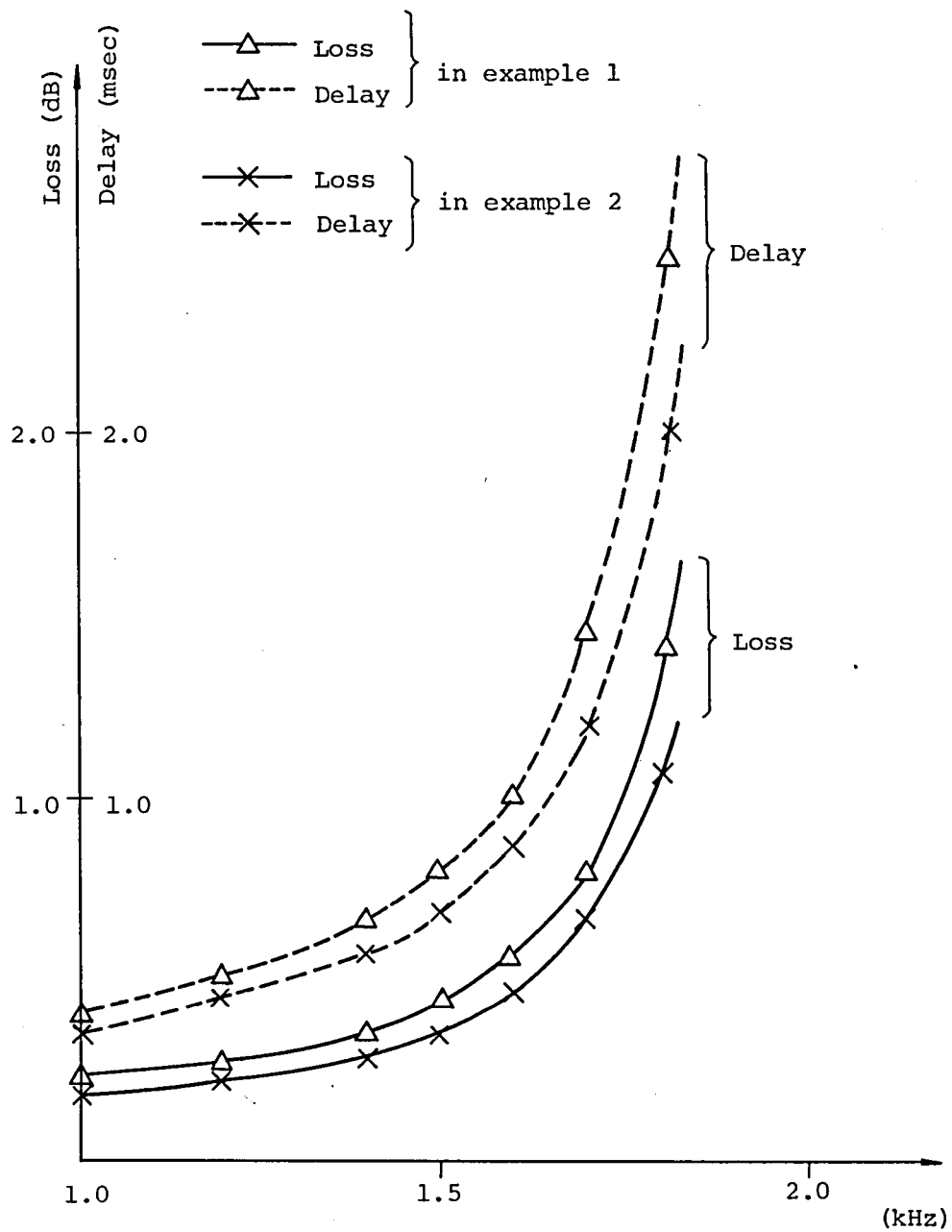


Fig. 2-11 The comparison with example 1 and example 2 for passband loss and delay characteristics with actual Q value at the vicinity of passband edge frequency.

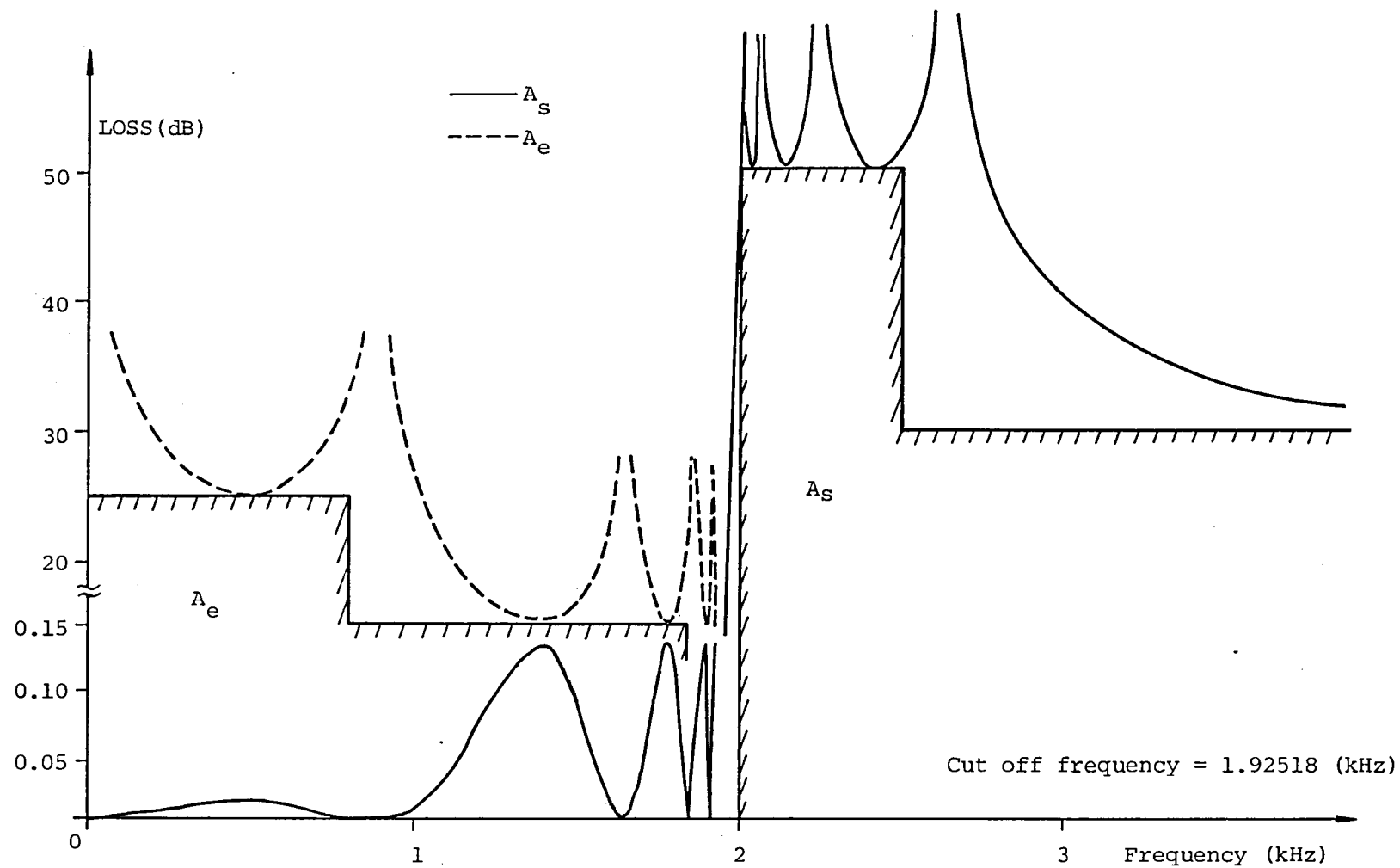


Fig. 2-12 Example 3 by the new method.

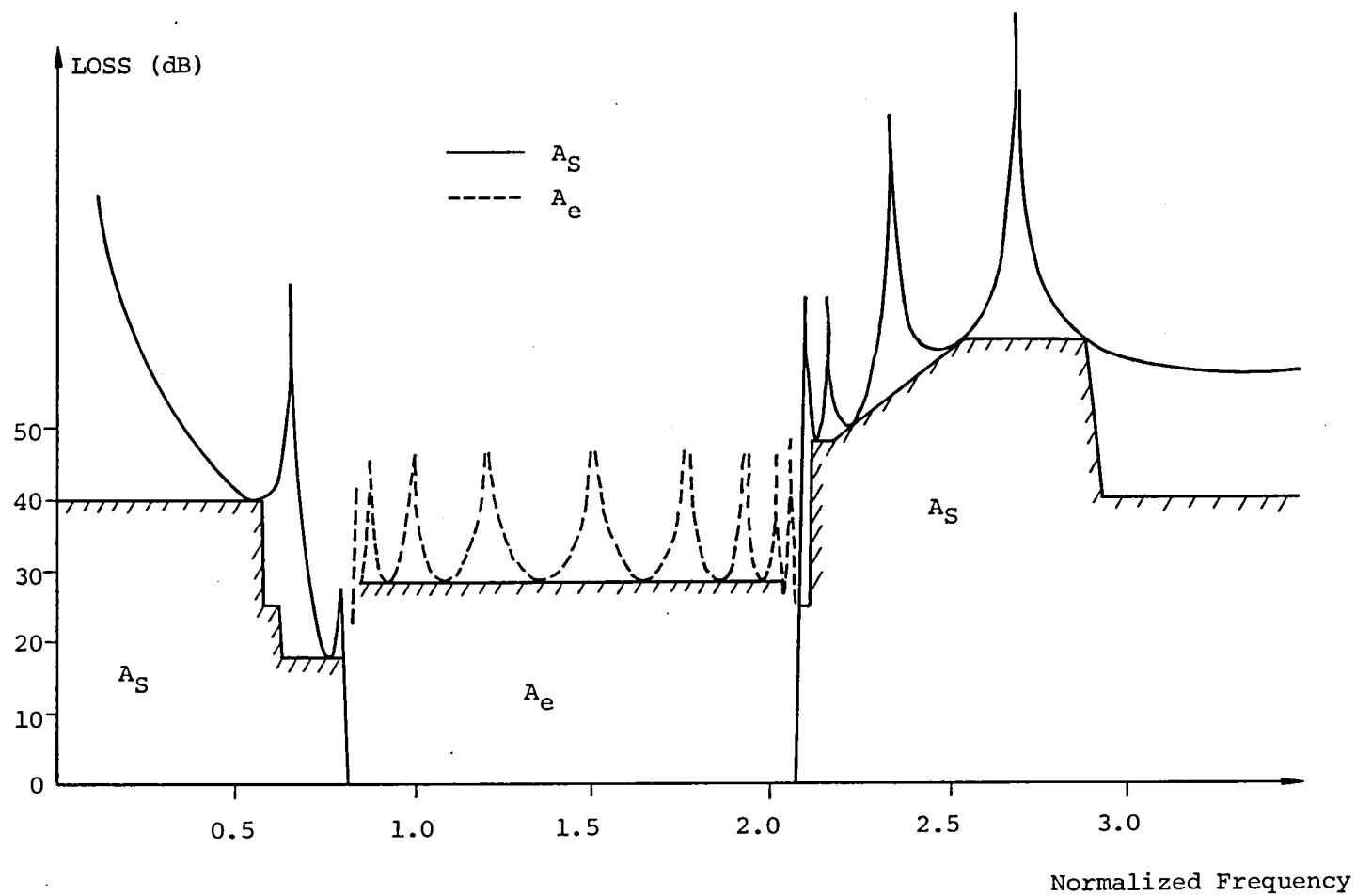


Fig. 2-13 Example 4 by the new method.

2-8 CONCLUSION

A computer aided approximation method by using filter characteristic function $\psi(p)$, which characterizes passband and stopband, was discussed. A method determining the optimum cut off frequency and the optimum filter degree, was introduced. Some examples show the excellency of this approximation theory by comparing with the conventional design method.

Furthermore, this method together with computer program can be fully applied to the digital filter design which will be described in the latter chapter.

CHAPTER 3. DESIGN OF REACTANCE BAND STOP FILTERS HAVING UNSYMMETRIC TRANSFER CHARACTERISTICS

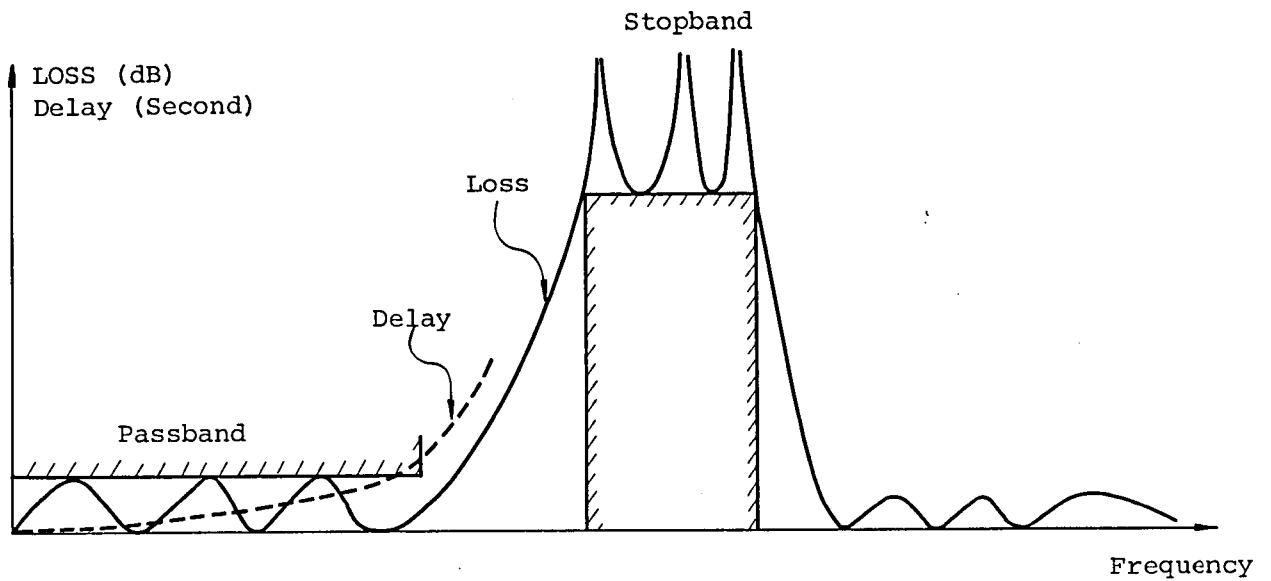
3-1 INTRODUCTION

Band stop filter (referred to as BSF) in communication systems can be used in the following two cases.

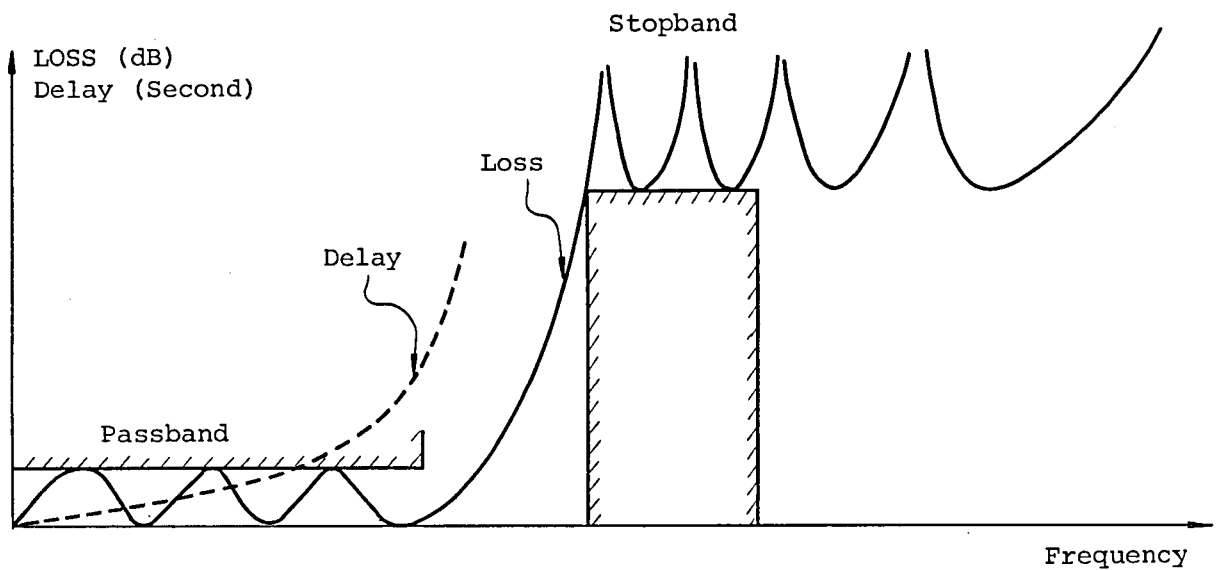
In the first case, a filter with a very narrow stopband in the signal frequency band and two passbands as wide as possible is required.

In the second case, a filter with a limited stopband width and one passband is required. In this case, BSF is sometimes used instead of LPF or HPF, because BSF does not have any attenuation pole at zero and infinity frequencies, which leads to a good group delay characteristic. Fig. 3-1 shows two realization methods by BSF and LPF comparing the group delay characteristics.

In many cases, BSF has been conventionally designed through the transformation of a prototype low pass filter. By using this method, the transfer characteristics of BSF becomes symmetric on the logarithmic frequency axis. Furthermore the attenuation poles at infinity in prototype low pass filter are transformed into multiple attenuation poles at center frequency of BSF. Therefore, when an unsymmetric attenuation characteristic is given as a specification, this BSF has an unreasonable attenuation characteristic which deteriorates the group delay characteristic and increases the total number of elements.



(i) Realization by BSF



(ii) Realization by LPF

Fig. 3-1 Two realizations by BSF and LPF for the same specifications with one passband and limited stopband width.

It is known that BSF having unsymmetric transfer characteristic in the stopband, can be designed by the image parameter method.⁽⁷⁾⁽⁸⁾ However, as mentioned earlier, this design method is rarely used in the actual stage. It should be noted here that we can apply the approximation theory by operating parameter method stated in Chapter 2 to the unsymmetric BSF design. In section 3-2, an approximation theory for unsymmetric BSF, will be briefly discussed. In section 3-3, a new realization theory for BSF will be proposed.

Many studies on realization techniques of three-terminal networks without using transformer, have been reported. Other than T. Fujisawa⁽⁹⁾, H. Watanabe⁽¹⁰⁾ mentioned in Chapter 1, H.C. Pande et al⁽¹¹⁾⁽¹²⁾ have given a realization condition for RC three-terminal network in the special case. T. Nishi⁽¹³⁾⁽¹⁴⁾⁽¹⁵⁾ has reported a realization condition for reactance three-terminal network. However, it is, in general, difficult to apply his method to an actual BSF realization because a large number of elements is required.

In this chapter a practical realization theory for BSF is proposed, which can reduce the number of circuit elements.⁽¹⁶⁾⁽¹⁷⁾

3-2 APPROXIMATION THEORY

BSF to be considered here is defined below. It is well known that $S(p)$ and $\psi(p)$ can be written as follows,⁽⁵⁾

$$S(p) = C \cdot H \frac{g(p)}{f(p)}$$

$$\psi(p) = H \frac{h(p)}{f(p)}$$

and, in a reactance network, we have a relation

$$S(p) \cdot S(-p) = 1 + \psi(p) \cdot \psi(-p).$$

[Definition 3-1]

$$(1) \quad S(j0) = S(j\infty) = 1$$

- (2) All the zeros of $f(p)$ whose degree N is even, exist on the $j\Omega$ axis, and all the zeros of $h(p)$ whose degree $M = N - 1$ is odd, exist on the $j\Omega$ axis.

BSF is defined so as to satisfy the conditions (1) and (2).

If we choose the degree of zeros in $h(p)$ at $j0$ as 1, (2) becomes the condition such that $\psi(p)$ has a largest number of parameters in approximation procedure. Definition 3-1 says that $\psi(p)$ in BSF has the form

$$\psi(p) = H \frac{p \cdot \prod_{i=1}^{(M-1)/2} (p^2 + C_i^2)}{\prod_{i=1}^{N/2} (p^2 + r_i^2)} \quad (3-1)$$

Here, the total number of parameters H , C_i and r_i becomes

$$1 + \frac{M-1}{2} + \frac{N}{2} = N.$$

It is clear that, by using $\psi(p)$, the approximation method

stated in Chapter 2 can be applied to BSF design. This can be easily seen in Fig. 3-2.

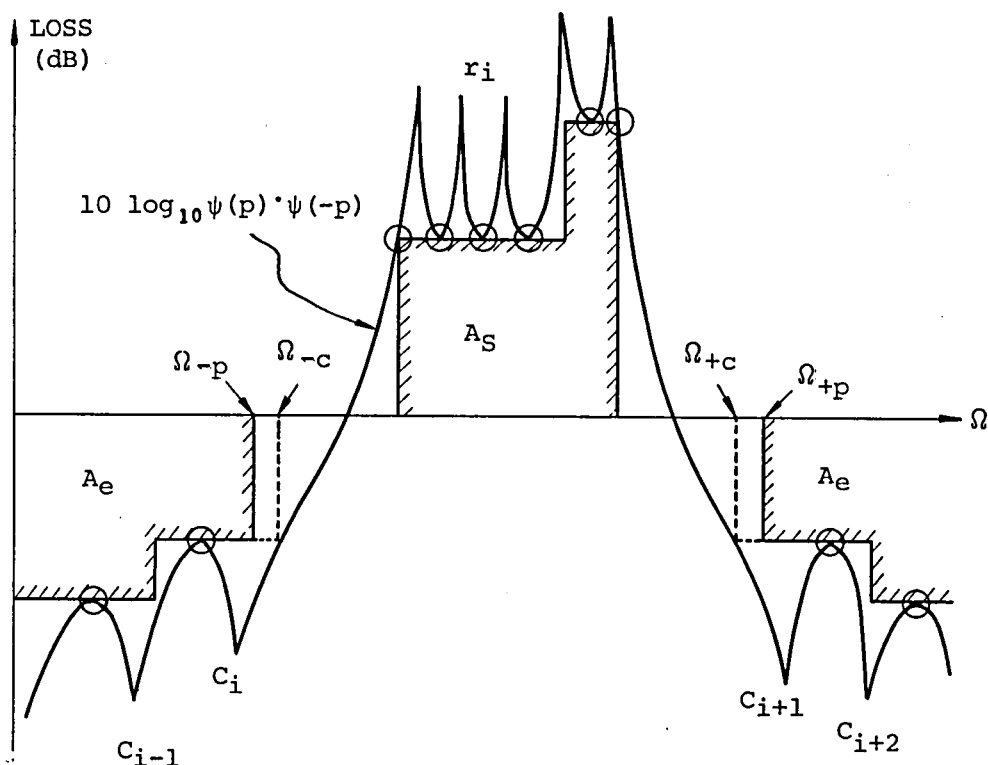


Fig. 3-2 Approximation by $\psi(p)$.

The cut off frequencies Ω_{-p} and Ω_{+p} given as specifications move to Ω_{-c} and Ω_{+c} where $\psi(p)$ has ten parameters and exactly meets the specifications at ten points indicated by circles in the figure.

3-3 REALIZATION THEORY

In this section, we introduce the sufficient condition for transformerless realization of BSF. Let us define the admittance matrix Y of reactance reciprocal two-terminal network.

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} .$$

Since the degree of $f(p)$ is even, Y can be written as⁽⁵⁾

$$Y = \begin{bmatrix} \frac{g_2(p)}{pU_1(p)} & -\frac{H^{-1}f(p)}{pU_1(p)} \\ -\frac{H^{-1}f(p)}{pU_1(p)} & \frac{g_1(p)}{pU_1(p)} \end{bmatrix} \quad (3-2)$$

where $g(p) = G(p) + U(p)$, $h(p) = G'(p) + U'(p)$,

$$g_1(p) = C \cdot G(p) + G'(p), \quad g_2(p) = C \cdot G(p) - G'(p),$$

$$pU_1(p) = C \cdot U(p) + U'(p).$$

$G(p)$ and $G'(p)$ express the even polynomials while $U(p)$ and $U'(p)$ express the odd polynomials. Since $h(p)$ is odd, $G'(p) = 0$. Therefore $g_1(p) = g_2(p)$ which is equivalent to $y_{11} = y_{22}$, and then it is called symmetric network. It is well known that this type of network can be realized by lattice circuits without use of transformer. In actual stage, however, it is rarely realized by lattice circuit and realization by three-terminal network with common earth is desired. A synthesis method by three-terminal network is introduced here, considering the property of BSF. y_{12} which is induced from $\psi(p)$ of Eq.(3-1), can be expressed by using Eq.(3-2) as follows:

$$-y_{12} = \frac{\prod_{n=1}^{N/2} (p^2 + r_n^2)}{A p \cdot \prod_{i=1}^L (p^2 + q_i^2) \cdot \prod_{j=L+1}^{(M-1)/2} (p^2 + q_j^2)}$$

where A is a positive constant. Without loss of generality, we can assume $r_n \leq r_{n+1}$, $q_i \leq q_{i+1}$ and $g_j \leq g_{j+1}$.

[Definition 3-2]

When the zeros and poles of y_{12} satisfy

$$q_1 \leq q_2 \leq \dots \leq q_L < r_1 \leq r_2 \leq \dots \\ \dots \leq r_{N/2} < q_{L+1} \leq q_{L+2} \leq \dots \leq q_{(M-1)/2},$$

we call that y_{12} satisfies the condition I.

The condition I expresses a general property in BSF because the attenuation poles which are equivalent to zeros of y_{12} , exist between two passbands which are characterized by q_i and q_j . We can write y_{12} in the partial fraction expansion expression.

$$-y_{12} = \frac{k_0}{p} + \sum_{i=1}^L \frac{k_i p}{p^2 + q_i^2} + \sum_{j=L+1}^{(M-1)/2} \frac{k_j p}{p^2 + q_j^2} + k_{\infty} p \quad (3-3)$$

[Lemma 3-1]

If y_{12} satisfies the condition I, we have

$$k_0 > 0, k_1 < 0, k_2 > 0, k_3 < 0, \dots, \text{ that is,} \\ \text{for even } i, k_i > 0, \text{ and for odd } i, k_i < 0.$$

We call these residues as low pass residues. In the same way, we have

$$k_{\infty} > 0, k_{(M-1)/2} < 0, k_{(M-3)/2} > 0, k_{(M-5)/2} < 0, \dots$$

that is, putting $m=(M-1)/2-j+1$, for even m , $k_m > 0$, and for odd m , $k_m < 0$. We call these residues as high pass residues.

(Proof) Low pass residues are calculated as follows:

$$k_0 = p(-y_{12}) \Big|_{p=0} = \frac{\prod_{n=1}^{N/2} (r_n^2)}{A \prod_{i=1}^L (q_i^2) \cdot \prod_{j=L+1}^{(M-1)/2} (q_j^2)} > 0$$

$$k_i = \frac{p^2 + q_i^2}{p} \cdot (-y_{12}) \Big|_{p^2 = -q_i^2}$$

$$= \frac{\prod_{n=1}^{N/2} (-q_i^2 + r_n^2)}{A (-q_i^2) (-q_i^2 + q_1^2) (-q_i^2 + q_2^2) \dots (-q_i^2 + q_{i-1}^2) (-q_i^2 + q_{i+1}^2) \dots} *$$

$$* \frac{\prod_{j=L+1}^{(M-1)/2} (-q_i^2 + q_j^2)}{\dots (-q_i^2 + q_L^2)}$$

In the above equation, since $q_i^2 < r_n^2$ for all n 's, the numerator is positive. In the denominator, each term of right side from $(-q_i^2 + q_{i+1}^2)$ is positive and each term of left side from $(-q_i^2 + q_{i-1}^2)$ is negative. For even i , or odd $i-1$, k_i is positive since the total number of negative terms is even. For odd i , k_i is negative.

High pass residues are calculated by

$$k_\infty = \frac{1}{p} (-y_{12}) \Big|_{p=\infty} = \frac{1}{A} > 0 .$$

Since $j = (M-1)/2 - m + 1$,

$$\sum_{j=L+1}^{(M-1)/2} \frac{k_j p}{p^2 + q_j^2} = \sum_{m=1}^{(M-1)/2-L} \frac{k_m p}{p^2 + q_m^2} .$$

Here, we have the relation $q_{m-1} \geq q_m$ and

$$k_m = \frac{p^2 + q_m^2}{p} (-y_{12}) \Big|_{p^2 = -q_m^2}$$

$$= \frac{\prod_{n=1}^{N/2} (-q_m^2 + r_n^2)}{A(-q_m^2 + q_1^2)(-q_m^2 + q_2^2) \cdots (-q_m^2 + q_{m-1}^2)(-q_m^2 + q_{m+1}^2) \cdots} *$$

$$* \frac{1}{\cdots (-q_m^2 + q_{(M-1)/2-L}^2)(-q_m^2) \prod_{i=1}^L (-q_m^2 + q_i^2)} .$$

Each term of numerator is negative. Since, in the denominator, each term of right side of $(-q_m^2 + q_{m+1}^2)$ is negative and each term of left side of $(-q_m^2 + q_{m-1}^2)$ is positive, we need to count up the total number of negative terms in the denominator. Here, we have the following table.

Table 3-1 Sign of high pass residue k_m

N/2	(M-1)/2	L	(M-1)/2-L	m	k_m
odd	even	odd	odd	even	positive
				odd	negative
		even	even	even	positive
				odd	negative
even	odd	odd	even	even	positive
				odd	negative
		even	odd	even	positive
				odd	negative

In table 3-1, we can easily see that for even m , k_m is positive and for odd m , k_m is negative. Q.E.D.

Lemma 3-1 says that if, in BSF, the zeros and poles of $-y_{12}$ are located as shown in Fig. 3-3, the sign of residues changes alternatively.

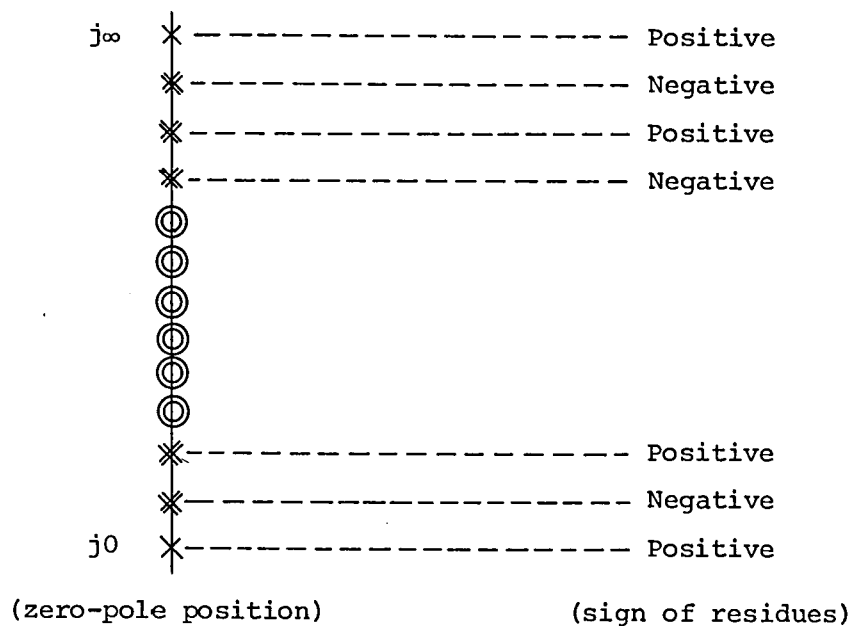


Fig. 3-3 Relation between zero-pole position and sign of residues.

In Fig. 3-3, \odot and \otimes mean second degree zeros and poles respectively, and \times means a first degree pole.

We will apply the result of Lemma 3-1 to the realization theory for BSF.

[Theorem 3-1]

In Eq.(3-3), if

$$k_0 > \sum_i |k_i| \quad \text{and} \quad k_\infty > \sum_j \left| \frac{k_j}{q_j^2} \right| ,$$

for all negative k_i

for all negative k_j

BSF can be realized by parallel connections of LPF, HPF and BPF.

(Proof) We divide Eq.(3-3) into two parts of low pass and high pass as $y_{12} = y_{12L} + y_{12H}$. First, we consider for y_{12L} only. Furthermore, y_{12L} is divided into two parts as follows:

$$-y_{12L} = \left(\frac{k_0}{p} + \sum_i \frac{k_i p}{p^2 + q_i^2} \right) + \left(\sum_i \frac{k_i p}{p^2 + q_i^2} \right)$$

for all negative k_i , for all positive k_i

$$= (-y_{12LA}) + (-y_{12LB}) .$$

Since, in $-y_{12LB}$, k_i is positive for all i , $-y_{12LB}$ can be realized by parallel connections of second order admittances. Each second order admittance consists of two elements. In $-y_{12LA}$, from the condition of the theorem, we can choose k_{0i} such that

$$k_{0i} > 0, k_{0i} > |k_i| \quad \text{and} \quad k_0 = \sum_i k_{0i} . \quad (3-4)$$

for all negative k_i

Therefore we can rewrite $-y_{12LA}$ as

$$-y_{12LA} = \sum_i \left(\frac{k_{0i}}{p} + \frac{k_i p}{p^2 + q_i^2} \right) .$$

for all negative k_i

For an arbitrary i , we have

$$\frac{k_{0i}}{p} + \frac{k_i p}{p^2 + q_i^2} = \frac{(k_{0i} + k_i) \left(p^2 + \frac{k_{0i} q_i^2}{k_{0i} + k_i} \right)}{p(p^2 + q_i^2)} . \quad (3-5)$$

Since $k_{0i} > |k_i|$, both $k_{0i} + k_i$ and $k_{0i} q_i^2$ are positive.

Hence, the zeros of Eq.(3-5) exist on the imaginary axis and are compared with q_i .

$$\frac{k_{0i} q_i^2}{k_{0i} + k_i} - q_i^2 = \frac{-k_i q_i^2}{k_{0i} + k_i} > 0$$

This relation satisfies the Fujisawa's theorem⁽⁹⁾ for the condition of realizing a low pass ladder filter by positive elements. This can be confirmed for all i in y_{12LA} . Therefore y_{12LA} can be realized by parallel connections of third degree low pass ladder networks. Each third degree network consists of four elements.

For y_{12H} , we can have the same discussions as y_{12L} .

As a result, y_{12} can be realized by the parallel connections of three types of network group, in which the first

group consists of parallel connections of third degree low pass ladder filters, the second group consists of parallel connections of third degree high pass ladder filters and the third group consists of parallel connections of second degree band pass filters.

Q.E.D.

As clearly shown in the above synthesis procedure, it is not necessary for residues to divide into low pass and high pass residues. By this consideration, we have the following corollary.

[Corollary]

When we rewrite y_{12} as

$$-y_{12} = \frac{k_0}{p} + \sum_{i=1}^{(M-1)/2} \frac{k_i p}{p^2 + q_i^2} + k_{\infty} p ,$$

and if we can have

$$k_0 + k_{\infty} > \sum_i |k_i| + \sum_i \left| \frac{k_i}{q_i^2} \right| ,$$

for some for other
negative k_i negative k_i

BSF can be realized by positive elements.

In the case of sixth degree or less degree BSF, the realization condition can be obtained by the relation of poles and zeros in y_{12} without using residues.⁽¹⁶⁾ y_{12} in sixth degree BSF can be written in the following form.

$$-y_{12} = \frac{(p^2+r_1^2)(p^2+r_2^2)(p^2+r_3^2)}{Ap(p^2+q_1^2)(p^2+q_2^2)} \quad (3-6)$$

[Theorem 3-2]

In Eq.(3-6), if

$$q_1 < r_1 \leq r_2 \leq r_3 < q_2 ,$$

sixth degree BSF can be realized by positive elements.

(Proof) y_{12} can be rewritten as follows:

$$-y_{12} = \frac{k_0}{p} + \frac{k_1 p}{p^2+q_1^2} + \frac{k_2 p}{p^2+q_2^2} + k_{\infty} p \quad .$$

From Lemma 3-1, we have $k_0 > 0$, $k_1 < 0$, $k_2 < 0$ and $k_{\infty} > 0$. Let us prove $k_0 > |k_1|$ and $k_{\infty} > |k_2/q_2^2|$. From Eq.(3-6), we can have k_0 and k_1 as follows:

$$\begin{aligned} k_0 &= \frac{r_1^2 r_2^2 r_3^2}{A q_1^2 q_2^2} \\ k_1 &= \frac{(-q_1^2+r_1^2)(-q_1^2+r_2^2)(-q_1^2+r_3^2)}{A(-q_1^2)(-q_1^2+q_2^2)} \\ \frac{k_0}{|k_1|} &= \frac{r_1^2 r_2^2 r_3^2 (q_2^2 - q_1^2)}{q_2^2 (r_1^2 - q_1^2) (r_2^2 - q_1^2) (r_3^2 - q_1^2)} \\ &= \frac{(1 - (\frac{q_1}{q_2})^2)}{q_2} \\ &= \frac{(1 - (\frac{q_1}{r_1})^2) (1 - (\frac{q_1}{r_2})^2) (1 - (\frac{q_1}{r_3})^2)}{q_2} \quad (3-7) \end{aligned}$$

From the condition of Theorem 3-2,

$$\left(\frac{q_1}{q_2}\right)^2 < \left(\frac{q_1}{r_3}\right)^2 \leq \left(\frac{q_1}{r_2}\right)^2 \leq \left(\frac{q_1}{r_1}\right)^2 < 1$$

and therefore,

$$\left(1 - \left(\frac{q_1}{q_2}\right)^2\right) > \left(1 - \left(\frac{q_1}{r_3}\right)^2\right) \geq \left(1 - \left(\frac{q_1}{r_2}\right)^2\right) \geq \left(1 - \left(\frac{q_1}{r_1}\right)^2\right) \quad (3-8)$$

Finally, substituting Eq.(3-8) into Eq.(3-7), we can obtain $k_0 > |k_1|$. Through almost the same discussions, we also have $k_\infty > |k_2/q_2^2|$. Q.E.D.

Therefore, sixth degree BSF can be realized by parallel connections of a low pass third degree ladder filter and a high pass third degree ladder filter.

3-4 CONSIDERATION ON NUMBER OF REQUIRED ELEMENTS

In an actual hardware realization stage, it is desired that the number of required elements is as small as possible. The relation between the filter degree and its number of required elements is investigated in detail. Since the filter degree N is even, we let $N = 2t$. Let us consider the two cases for odd t and even t . For odd t , since the total number of residues excluding k_0 and k_∞ is even, there exist the following three cases.

(case 1) In Eq.(3-3), all the residues excluding k_0 and k_∞

are only low pass residues or only high pass residues.

In this case, since the number of negative residues is $(t-1)/2$ and the number of elements required for each negative residue is 4, the number of elements required for all negative residues becomes $2(t-1)$. Since the number of positive residues is also $(t-1)/2$ and each required element number is 2, the number of elements for all positive residues becomes $t-1$. For k_0 or k_∞ , one element is needed. Therefore, in the case 1, the total number of elements becomes $3t-2$.

(case 2) The number of low pass residues excluding k_0 is even and the number of high pass residues excluding k_∞ is odd.

Since the number of negative residues and the number of positive residues are equally $(t-1)/2$, the total number of elements becomes $3t-3$.

(case 3) The number of low pass residues excluding k_0 is odd and the number of high pass residues excluding k_∞ is odd.

Since the number of negative residues is $(t+1)/2$ and the number of positive residues is $(t-3)/2$, the total number of elements becomes $3t-1$.

For even t , the total number of residues excluding k_0 and k_∞ is odd. Through the same discussions as the case of odd t , the total number of elements becomes $3t-1$ for case 1 and $3t-2$ for cases 2 and 3.

Table 3-2 shows the relation between N and the total number of elements.

Table 3-2 Relation between N and the total number of elements

N \ Case	2	4	6	8	10	12	14	16	18	20
(1)	2	5	7	11	13	17	19	23	25	29
(2)				10	12	16	18	22	24	28
(3)			8	10	14	16	20	22	26	28

3-5 DESIGN EXAMPLES

3-5-1 Example 1 of 12th degree BSF designed by the new method

$$\begin{aligned}
 -Y_{12} &= \frac{\prod_{i=1}^6 (p^2 + r_i^2)}{AP \prod_{i=1}^5 (p^2 + q_i^2)} \\
 &= \frac{k_0}{p} + \sum_{i=1}^5 \frac{k_i p}{p^2 + q_i^2} + k_{\infty} p
 \end{aligned}$$

r_1^2	0.415907	A	1.699730	k_0	0.356321
r_2^2	0.527006	q_1^2	0.299696	k_1	-0.098655
r_3^2	0.611123	q_2^2	0.354565	k_2	0.024646
r_4^2	0.725299	q_3^2	0.945084	k_3	-0.030215
r_5^2	0.806308	q_4^2	0.960701	k_4	0.057644
r_6^2	0.909870	q_5^2	1.219778	k_5	-0.182845
				k_{∞}	0.588328

Example 1 satisfies the conditions of Lemma 3-1 and Theorem 3-1. Therefore, we have

$$-Y_{12L} = \frac{k_0}{p} + \frac{k_1 p}{p^2 + q_1^2} + \frac{k_2 p}{p^2 + q_2^2}$$

$$-Y_{12H} = \frac{k_3 p}{p^2 + q_3^2} + \frac{k_4 p}{p^2 + q_4^2} + \frac{k_5 p}{p^2 + q_5^2} + k_{\infty} p$$

Assuming that the residues are compact, y_{11} can be expressed as

$$Y_{11L} = \frac{k_0}{p} + \frac{|k_1| p}{p^2 + q_1^2} + \frac{k_2 p}{p^2 + q_2^2}$$

and

$$Y_{11H} = \frac{|k_3| p}{p^2 + q_3^2} + \frac{k_4 p}{p^2 + q_4^2} + \frac{|k_5| p}{p^2 + q_5^2} + k_{\infty} p$$

where $Y_{11} = Y_{11L} + Y_{11H}$. BSF can be realized by parallel connections of LPF using k_0 and k_1 , HPF (high pass filter) using k_3 , k_5 and k_{∞} , and BPF using k_2 and k_4 . Network configuration and element values are shown in Fig. 3-4. Loss characteristics are shown in Fig. 3-5.

3-5-2 Example 2 of 6th degree BSF designed by the new method

$$-Y_{12} = \frac{(p^2 + 0.416658)(p^2 + 1.217391)(p^2 + 1.629783)}{2.926205p(p^2 + 0.254194)(p^2 + 3.386999)}$$

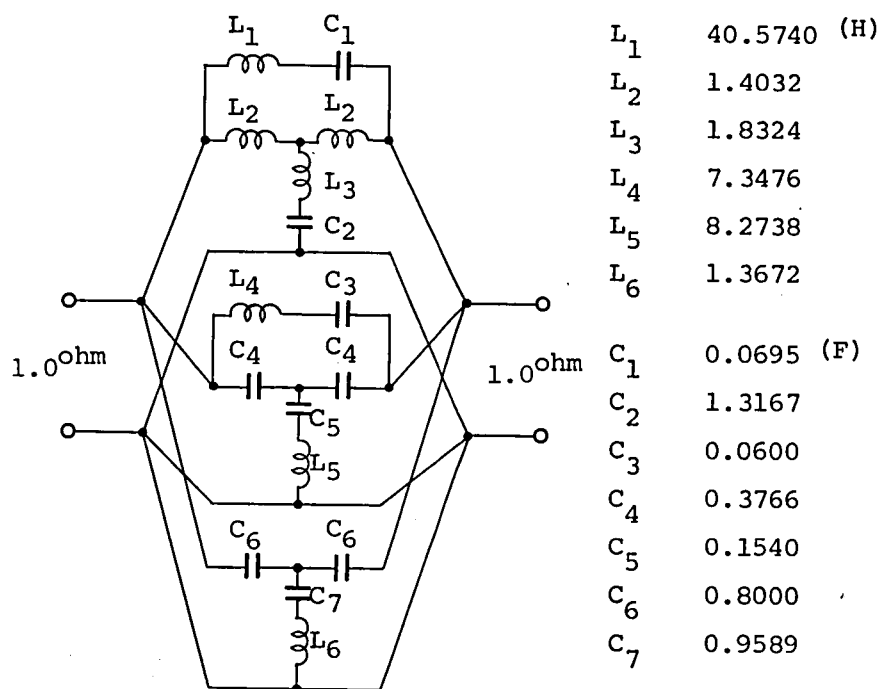


Fig. 3-4 Network configuration and element values.
(Example 1)

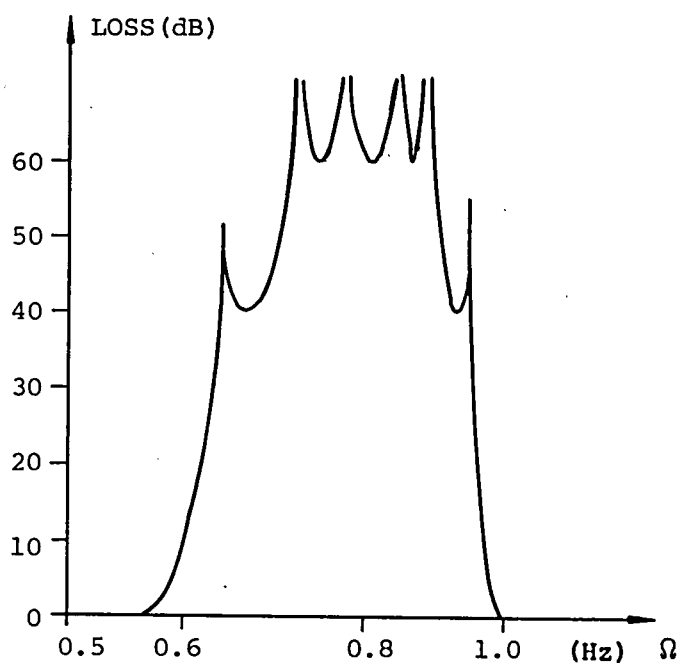


Fig. 3-5 Loss characteristic. (Example 1)

Example 2 satisfies the condition of Theorem 3-2. Network configuration and element values are shown in Fig. 3-6. Loss and delay characteristics are shown together with loss specification in Fig. 3-7.

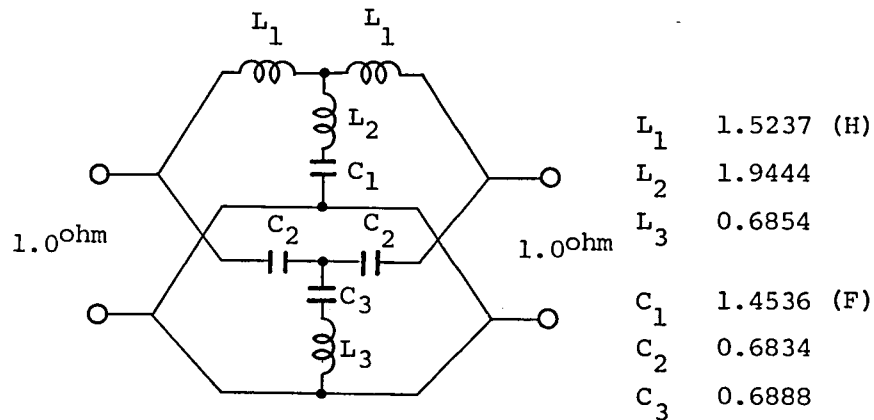


Fig. 3-6 Network configuration and element values. (Example 2)

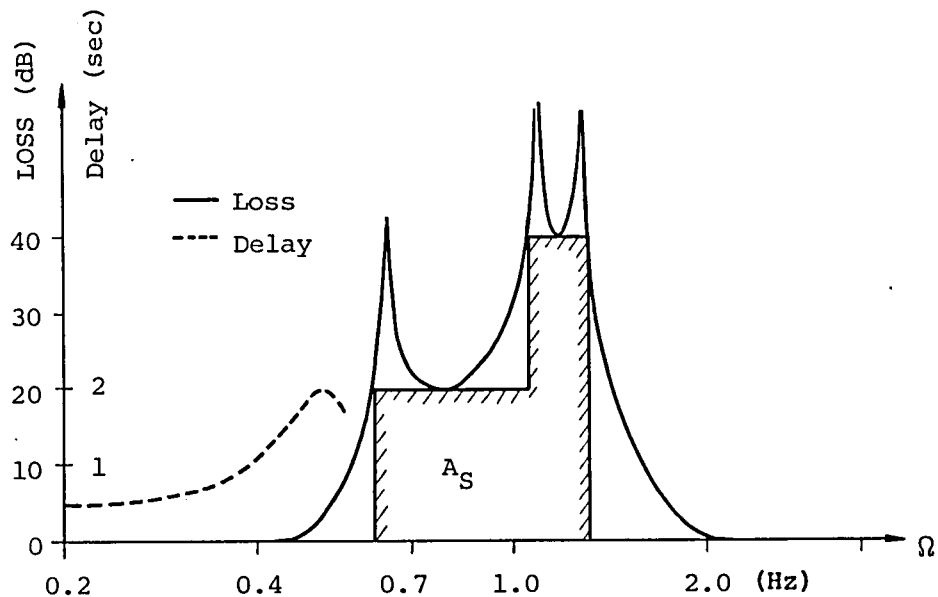


Fig. 3-7 Delay and loss characteristics. (Example 2)

3-5-3 Example 3 of 10th degree BSF designed by the conventional method

Example 3 is designed for the same specification as example 2 through the reactance transformation of prototype LPF. Network configuration and element values are shown in Fig. 3-8. Loss and delay characteristics are shown together with specification in Fig. 3-9. This BSF has three multiple attenuation poles at center frequency and two symmetrical attenuation poles as shown in Fig. 3-9.

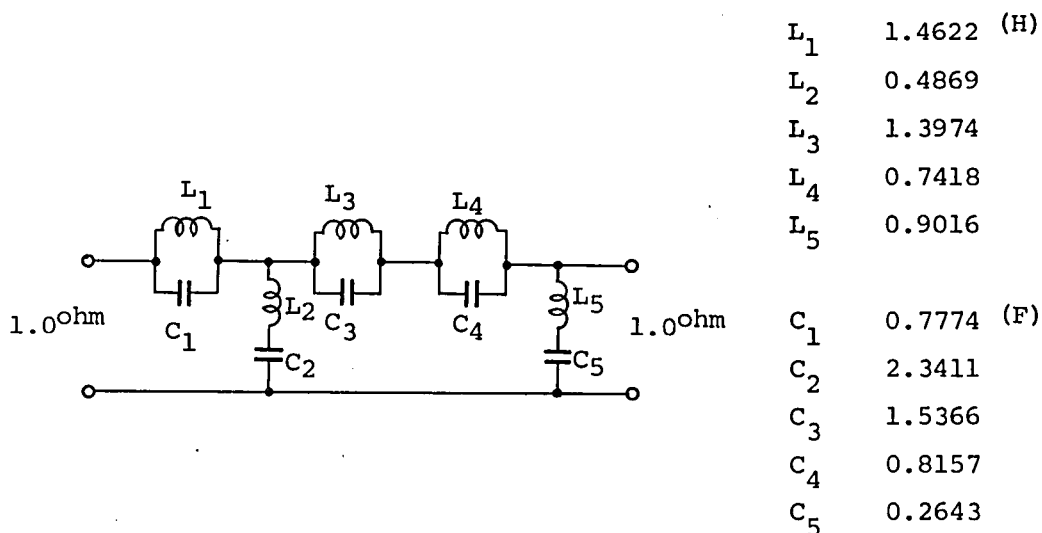


Fig. 3-8 Network configuration and element values.
(Example 3)

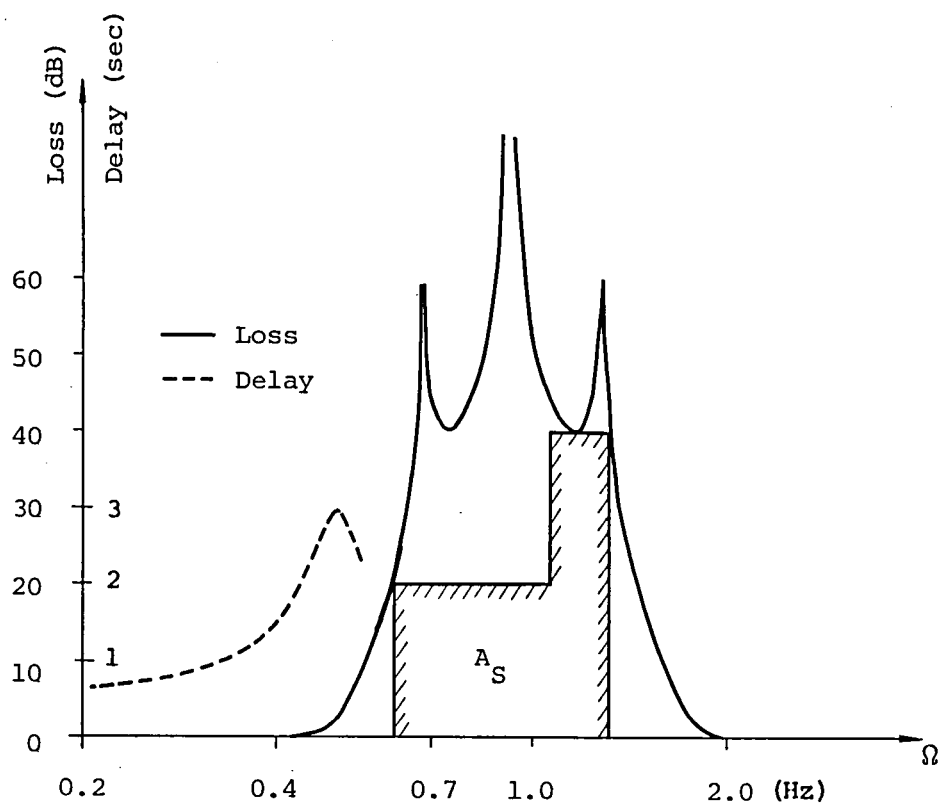


Fig. 3-9 Delay and loss characteristics. (Example 3)

3-5-4 Comparison of examples 2 and 3

In the delay characteristics, example 2 is superior to example 3. In the number of elements, example 2 needs 8 elements and, on the other hand, example 3 requires 10 elements.

3-6 CONCLUSION

A design theory for BSF with unsymmetric transfer characteristics was described. Lemma 3-1 shows a general property of the residues in BSF. Based on this property, a practical

realization theory together with synthesis procedures was given. In addition, the necessary number of elements which becomes a problem in an actual design stage was examined and it has been shown that the number of elements is less than one and a half times of filter degree. Finally, it was shown that BSF designed by using this method is excellent compared with the one designed by the conventional method, in the delay characteristic and the number of elements.

The ladder realization which is considered to be most practical in the manufacturing stage, will become a next problem while we developed here the parallel connection realization by means of admittance matrix.

CHAPTER 4. COMPUTER AIDED DESIGN OF THE NORTON TRANSFORMATION ON REACTANCE BAND PASS LADDER FILTERS

4-1 INTRODUCTION

In this chapter, a computer aided design method of the equivalent circuit transformation is described, which is one of a design process on reactance band pass ladder filters (referred to as BPF in this chapter). The design of BPF by operational parameter method, is usually carried out through the following processes⁽⁵⁾.

- (1) Determine a transfer function satisfying a specification.
- (2) Synthesize a ladder circuit through making a cascade matrix from the transfer function.
- (3) Determine optimum element values through equivalent circuit transformation.
- (4) Simulate various characteristics by approximating an actual hardware implementation.

In the above four processes, the computer aided design methods for (1), (2) and (4), have been successfully developed. The process (3) called Norton transformation⁽⁵⁾⁽¹⁸⁾⁽¹⁹⁾⁽²⁰⁾ implies that the element values obtained by process (2) have usually a problem for an actual realization and, therefore, they have to be transformed into easily realizable element values which

are, as usual, so chosen that their inductances have optimum values for Q characteristics.

Since we have no systematic transformation process and, therefore, it is difficult to predict the final network topology after the Norton transformation for an original network topology given in an output of process (2), it is very hard to organize CAD program system for this process. Usually, this process has been carried out by man-machine interaction using graphic display.⁽¹⁹⁾⁽²⁰⁾ This procedure is repeated until the reasonable element values are obtained. However, the effectiveness of the indication by a filter designer, considerably depends on the designer's filter design career and, even if he has lots of career, this process requires a lot of time consuming work. Furthermore, this process prevents from organizing the total design automation system of BPF.

In this chapter, a method carrying out the Norton transformation automatically by using a heuristic optimization procedure, without depending upon a designer's career, is described.⁽²¹⁾⁽²²⁾ By this new method, we can organize the automatic BPF design program system throughout (1)-(4) processes.

4-2 NORTON TRANSFORMATION

The object of the Norton transformation is (i) to transform the original element values given in process (2) into easily realizable element values and (ii) to eliminate an ideal transformer (transformer ratio is given by $1:T$) being included

in the network obtained by process (2).*

Since BPF is usually realized by a network of minimum number of coils, we can restrict the following discussions to the condenser transformation. We make an abbreviation of condenser and coil as C and L, respectively. A condenser transformation is defined by the equivalent circuit transformation of two C's and one ideal transformer as shown in Figs. 4-1-a and 4-1-b.

In Fig. 4-1-a, the condition that all the element values obtained by the transformation are positive, is

$$1 < \phi_i < 1 + \frac{C_2}{C_1} \quad (4-1)$$

and, in Fig. 4-1-b,

$$\frac{1}{1 + \frac{C_2}{C_1}} < \phi_i < 1 \quad (4-2)$$

An arbitrary number n of ideal transformer ϕ_i ($i=1, 2, \dots, n$) which expresses each transformer ratio, can be put in the circuit. Here, we have the restriction of

*foot note - If we can choose arbitrary input and output impedances, we can determine their impedances so that the transformer is exactly eliminated. Otherwise, there exists an ideal transformer in the network⁽¹⁰⁾.

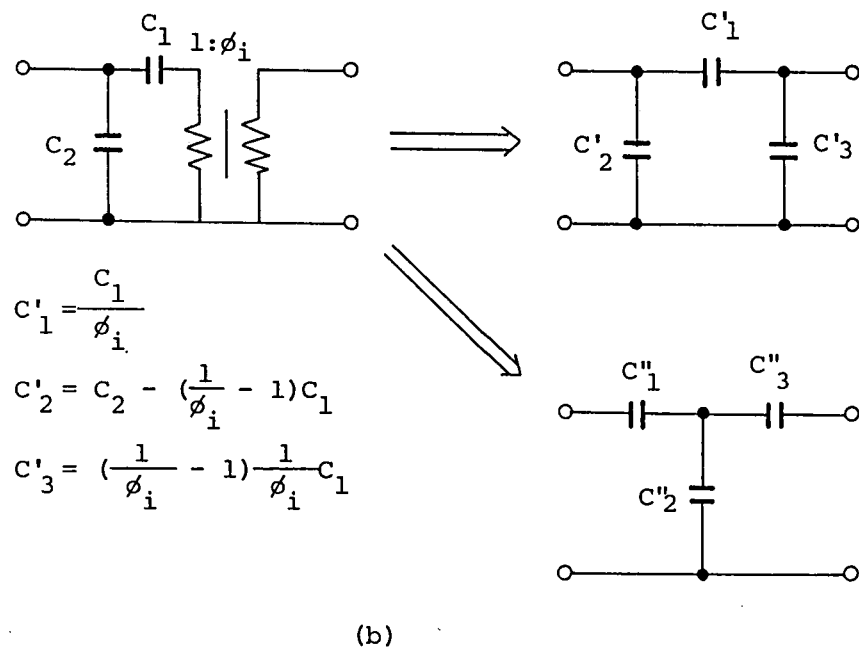
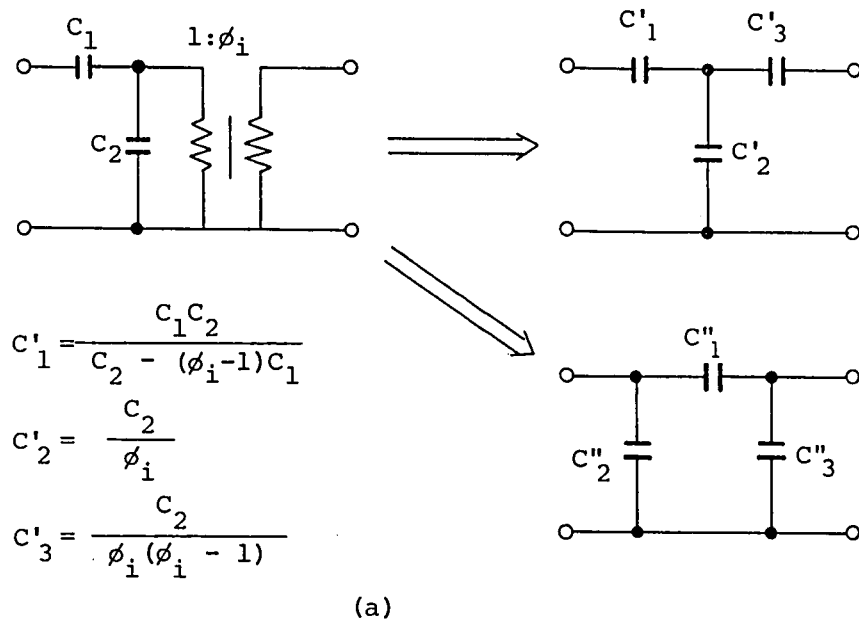


Fig. 4-1 Norton transformation.

$$\phi_1 \cdot \phi_2 \cdot \dots \cdot \phi_n \cdot T = \sqrt{\frac{R_{out}}{R_{in}}} \quad (4-3)$$

where R_{in} and R_{out} express the input and output impedance, respectively. For given values of R_{in} , R_{out} and T , if we choose arbitrary values of ϕ_1 , ϕ_2 , \dots , ϕ_{n-1} , ϕ_n can be uniquely determined by Eq.(4-3). The above mentioned process is called Norton transformation.

In the network after the Norton transformation, we can have the equivalent circuit transformation for each arm as shown in Fig. 4-2.

It should be noted here that this transformation can be carried out independently for each arm. This process is called 2-terminal impedance transformation.

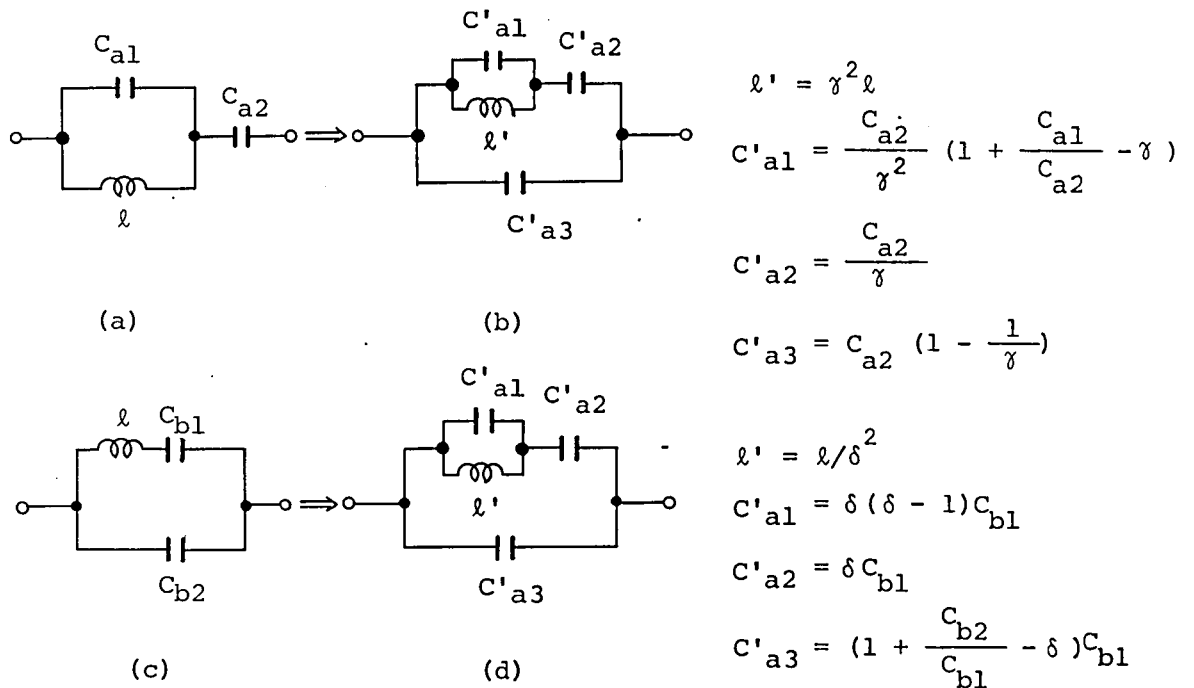


Fig. 4-2 2-terminal impedance transformation.

4-3 NETWORK TOPOLOGY EXPRESSION

Since the Norton transformation program is not only used as a subroutine of perfectly automated BPF design program but also used as an independent main program, it is desired that a network topology is easily expressed for programming and also the necessary amount of program input data is as small as possible. We can restrict the BPF topology expression to the following six network configurations⁽¹⁰⁾.

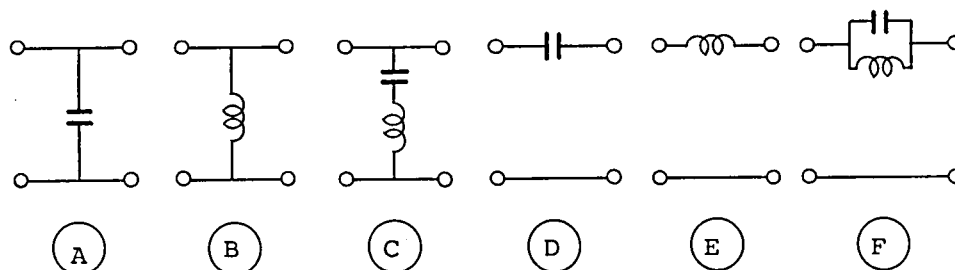


Fig. 4-3 Six network configurations of BPF.

All the BPF obtained from process (2) can be expressed by the cascade connection of (A) - (F) networks, in which the order of network connection is arbitrary. Therefore, the input data in the program require only the (A) - (F) connection order and their element values with T.

In this expression, if arbitrary two networks out of (A) (B) (C) are successively connected, we can change their connection order. We have the same things for (D) (E) (F).

The details of topology expression examples will be shown in the Appendix.

4-4 CONSIDERATION ON EVALUATION FUNCTION

The problem is what the optimum equivalent circuit is. Strictly speaking, there are different solutions for this problem, for each cases when considering several situations such as the frequency range to be used, filter volume to be required and cost etc. However, from the actual realization standpoint, it is natural to consider the following three evaluations.

- (1) The total number of required elements is as small as possible.
- (2) Each element exists within possibly realizable values.
- (3) Inductance has a value such that Q of a coil is high at the frequency considered.

By increasing the number of the ideal transformer to be put in the circuit and increasing the 2-terminal impedance transformation process, the total number of C increases. Since, however, C is, in general, less costly compared with L and inductance is a more important factor for performances, we may neglect the evaluation (1) here.

A volume of filter with extremely large capacitance becomes big and a characteristic of filter with extremely small capacitance is affected by some parasitic elements. On the evaluation (2), therefore, we give a restriction of upper limit C_u and lower limit C_l for capacitance.

Through BPF synthesis procedure (process (2) in section 4-1), the attenuation poles at infinite frequency and at zero frequency (called n_{∞} and n_0 respectively) are, in general, arranged at input and output of BPF network⁽¹⁰⁾. Therefore, the inductances constituting these attenuation poles are not changed by both the Norton transformation and the 2-terminal impedance transformation. These element values are determined by the given input and output impedances. Here we have the following two cases on the evaluation (3). First is the case that we can choose the input and output impedances so that the inductances constituting n_{∞} and n_0 have values giving high Q characteristics. Second is the case that the input and output impedances are both given as a constant. In this case, the inductances constituting n_{∞} and n_0 do not often have the optimum values for Q characteristics. However, due to the fact that L's constituting attenuation poles at finite frequencies sensitively affect the frequency characteristics while L's constituting n_{∞} and n_0 have small sensitivity, we may neglect the contributions of n_{∞} and n_0 . Therefore, for both cases, we can have the following evaluation function.

$$E = \max_i |L_i - L_I| \quad (4-4)$$

where L_i expresses inductance of L constituting attenuation pole at finite frequency and L_I means an ideal inductance value for Q characteristics. For the first case, L_I is given as an

average value of inductances constituting n_{∞} and n_0 , and, for the second case, L_I is given by input data. Here, the problem is to find a circuit minimizing E within the given range of C_u and C_l .

4-5 ALGORITHM

4-5-1 Determination of the number n and the positions of ideal transformers

First, we determine the number n of ideal transformer to be put in the circuit, where n is given by the random number generation. Assume that an ideal transformer can be put in the circuit in pairs of circuit (A) or circuit (D) in Fig. 4-3. Because we, in general, can not have the network as shown in Fig. 4-1 in an original circuit obtained by process (2), we can not make a combination of one ideal transformer and two C's. Consequently, the total sum K of (A) and (D) becomes maximum possible number for ideal transformer. Multiplying K by the uniformly distributed random number generated between $0 \sim 1.0$ and then rounding it, we obtain an integer value n lower than K .

Next, we determine which (A)'s or (D)'s should be paired with an ideal transformer, that is, the positions of n ideal transformers to be put in the original circuit are determined by generating random numbers. This pairing process is called here that 'C is specified' for abbreviation. We make

the numbering for (A) and (D), in order, from an input in the original circuit. We generate n different integer values in the integer values from 1 to K, which give the n different positions. In this way, we have N_1 sets of equivalent circuits with different configuration.

In some cases, there exists two adjacent (A) and (D) paired with each ideal transformer. In Fig. 4-1-a, only Π type circuit is permitted for transformation, and in Fig. 4-1-b, T type circuit is permitted so that we can do the Norton transformation for both (A) and (D). In other words, when C_1 and C_2 in Fig. 4-1-a are both specified, we make Π type transformation for C_1 and then we make T type transformation for C_2 in Π type circuit instead of C_2 . Similarly, when C_1 in Fig. 4-1-b is specified, this specification is held in C_2 in T type circuit.

4-5-2 Confirmation of the Norton Transformation Possibility

Here we confirm the Norton transformation possibility. When C_1 or C_2 is specified, we confirm, in order, from the input of the circuit whether its specified C has (A) (D) or (D) (A) connection. If its specified C does not have (A) (D) or (D) (A) connection, we look at the next adjacent network of (A) or (D). When the next adjacent network of (A) or (D) is changeable, we change them and then, confirm again its possibility. If, after doing this, we can not yet find its possibility, we proceed the next stage Norton transformation

but hold the pre-stage specification. This process will be explained together with two examples in the Appendix.

4-5-3 Determination of Transformer Ratio

ϕ_i ($i = 1, 2, \dots, n-1$) satisfying Eq.(4-1) or Eq.(4-2), is determined by generating random number. In this process, we have N_2 sets of equivalent circuits with different element values for each set of ϕ_i . Element values transformed by ϕ_i ($i = 1, 2, \dots, n-1$) are all positive, while ϕ_n determined by Eq.(4-3) does not always satisfy Eq.(4-1) or Eq.(4-2). Therefore, we choose some sets from N_2 sets where ϕ_n satisfies Eq.(4-1) or Eq.(4-2).

4-5-4 2-Terminal Impedance Transformation

For some sets of equivalent circuits chosen in Section 4-5-3, we carry out the 2-terminal impedance transformation. This transformation can be independently done for each arm in the circuits. Here we take a consideration on C_u and C_ℓ . First, we consider to transform Fig. 4-2-a circuit into Fig. 4-2-b circuit. The following inequalities hold.

$$0 < C_\ell \leq C'_{a1} = \frac{C_{a2}}{\gamma^2} \left(1 + \frac{C_{a1}}{C_{a2}} - \gamma\right) \leq C_u \quad (4-5)$$

$$0 < C_\ell \leq C'_{a2} = \frac{C_{a2}}{\gamma} \leq C_u \quad (4-6)$$

$$0 < C_{\ell} \leq C'_{a2} = \frac{C_{a2}}{\gamma} (\gamma - 1) \leq C_u \quad . \quad (4-7)$$

From the above three inequalities, a possible range of γ is given. From Eq.(4-5), we have the second degree equations on γ ,

$$C_{\ell}\gamma^2 + C_{a2}\gamma - (C_{a1} + C_{a2}) = f_{\ell}(\gamma) \leq 0$$

and (4-8)

$$C_u\gamma^2 + C_{a2}\gamma - (C_{a1} + C_{a2}) = f_u(\gamma) \geq 0 \quad .$$

Both $f_{\ell}(\gamma)$ and $f_u(\gamma)$ have one positive root and one negative root. Since γ must be positive, the range of γ is determined by

$$\gamma_{\ell} \leq \gamma \leq \gamma_u$$

where

$$\gamma_{\ell} = \frac{-C_{a2} + \sqrt{C_{a2}^2 + 4C_u(C_{a1} + C_{a2})}}{2C_u}$$

and (4-9)

$$\gamma_u = \frac{-C_{a2} + \sqrt{C_{a2}^2 + 4C_{\ell}(C_{a1} + C_{a2})}}{2C_{\ell}} \quad .$$

Fig. 4-4 shows the range of γ .

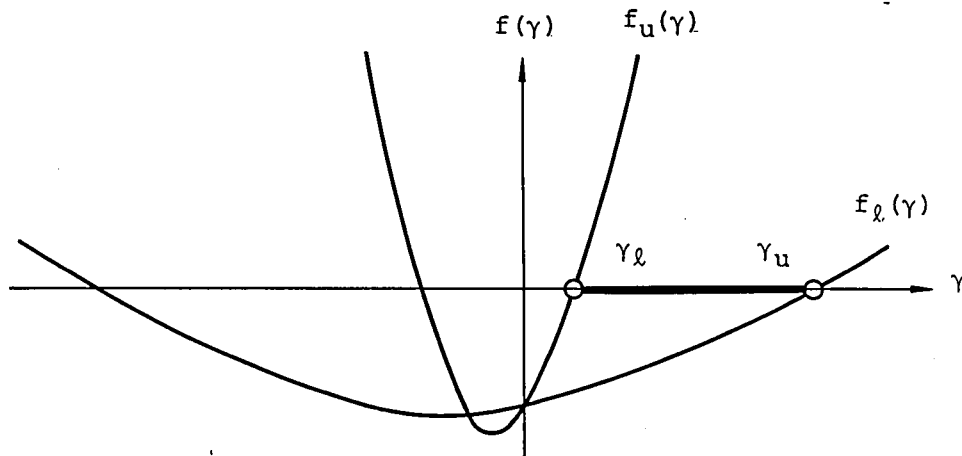


Fig. 4-4 The range of γ satisfying Eq. (4-5).

From Eq. (4-6), we have

$$\frac{C_{a2}}{C_u} \leq \gamma \leq \frac{C_{a2}}{C_l} \quad (4-10)$$

and, from Eq. (4-7),

$$C_{a2} \leq \gamma(C_{a2} - C_l) \quad (4-11)$$

and

$$\gamma(C_{a2} - C_u) \leq C_{a2} \quad (4-12)$$

For Eqs. (4-11) and (4-12), there exist the following three cases.

$$(1) \quad C_{a2} < C_l$$

Since the transformed element values becomes lower than C_l , this case can be omitted.

$$(2) \quad C_{\ell} \leq C_{a2} \leq C_u$$

From Eq.(4-11), we have

$$\frac{C_{a2}}{C_{a2} - C_{\ell}} \leq \gamma \quad .$$

Eq.(4-12) is satisfied. $K_{\min 0}$ and $K_{\max 0}$ are defined as follows:

$$K_{\min 0} = \max \left[\gamma_{\ell}, \frac{C_{a2}}{C_u}, \frac{C_{a2}}{C_{a2} - C_{\ell}} \right]$$

$$K_{\max 0} = \min \left[\gamma_u, \frac{C_{a2}}{C_{\ell}} \right] \quad .$$

Using the above definition, we have the following relation for a possible range of γ ,

$$K_{\min 0} \leq \gamma \leq K_{\max 0}$$

$$(3) \quad C_u < C_{a2}$$

Combining Eq.(4-11) and Eq.(4-12), we have

$$\frac{C_{a2}}{C_{a2} - C_{\ell}} \leq \gamma \leq \frac{C_{a2}}{C_{a2} - C_u} \quad .$$

$K_{\max 1}$ is defined as follows:

$$K_{\max 1} = \min \left[\gamma_u, \frac{C_{a2}}{C_{\ell}}, \frac{C_{a2}}{C_{a2} - C_u} \right] \quad .$$

Therefore, γ has a range of

$$K_{\min 0} \leq \gamma \leq K_{\max 1} \quad .$$

Similarly, for the transformation from Fig. 4-2-c circuit to Fig. 4-2-d circuit, a possible range of δ can be determined according to the following three cases.

$$(1) \quad C_{b1} + C_{b2} < C_{\ell}$$

Since the transformed element values becomes lower than C_{ℓ} , this case can be omitted.

$$(2) \quad C_u < C_{b1} + C_{b2}$$

$$K'_{\min 0} \leq \delta \leq K'_{\max 0}$$

where

$$K'_{\min 0} = \max \left[\frac{C_{\ell}}{C_{b1}}, \delta_{\ell}, 1 + \frac{1}{C_{b1}} (C_{b2} - C_u) \right]$$

$$K'_{\max 0} = \min \left[\frac{C_u}{C_{b1}}, \delta_u, 1 + \frac{1}{C_{b1}} (C_{b2} - C_{\ell}) \right]$$

and

$$\delta_{\ell} = \frac{C_{b1} + \sqrt{C_{b1}^2 + 4 C_{b1} C_{\ell}}}{2 C_{b1}}$$

$$\delta_u = \frac{C_{b1} + \sqrt{C_{b1}^2 + 4 C_{b1} C_u}}{2 C_{b1}}$$

$$(3) \quad C_\ell \leq C_{b1} + C_{b2} \leq C_u$$

$$K'_{min1} \leq \delta \leq K'_{max0}$$

where

$$K'_{min1} = \max \left[\frac{C_\ell}{C_{b1}}, \delta_\ell \right]$$

By the range of γ and δ , the range of ℓ' in Fig. 4-2 is given and then L_i satisfying Eq.(4-4) is uniquely determined for each arm. This will be clear in Fig. 4-5 which shows a case of three coils.

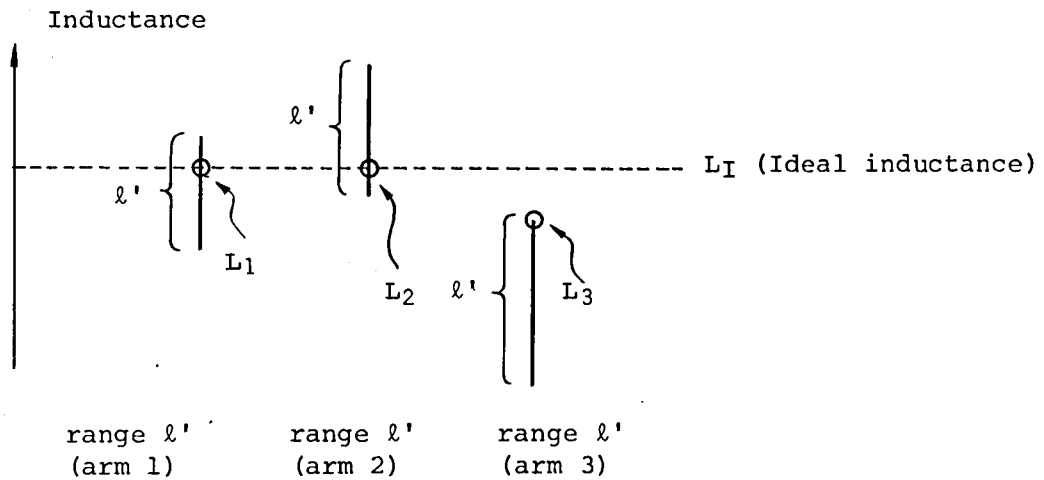


Fig. 4-5 Choice of L_i .

In Fig. 4-5, the values nearest to L_I are chosen as L_1 , L_2 and L_3 (indicated by circles).

This process is repeated for some sets of equivalent circuits and then, finally, we obtain one equivalent circuit having minimum E.

4-5-5 T Type-II type Circuit Transformation

When, through the above procedure, there still remains C values outside the range of C_l and C_u (called C_0), T type-II type circuit transformation should be carried out. In the case that $C_0 < C_l$ and C_0 is included in (A) (D) (A) or (A) (F) (A) network, by the II to T type circuit transformation, larger C values can be obtained. In the case that $C_0 > C_u$ and C_0 is included in (D) (A) (D) or (D) (C) (D) network, by the T to II type circuit transformation, smaller C values can be obtained. These circuit transformations are shown in Fig. 4-6.

4-6 EXAMINATION

We call here a product of N_1 and N_2 as a pattern generation number (PGN). One PGN coincides a circuit presenting a set of ideal transformer positions and their transformer ratio ϕ_i ($i=1, 2, \dots, n$). Fig. 4-7 shows a relation of PGN and the maximum or minimum L value (L_{max} , L_{min}) for various numbers of ideal transformer n as a parameter.

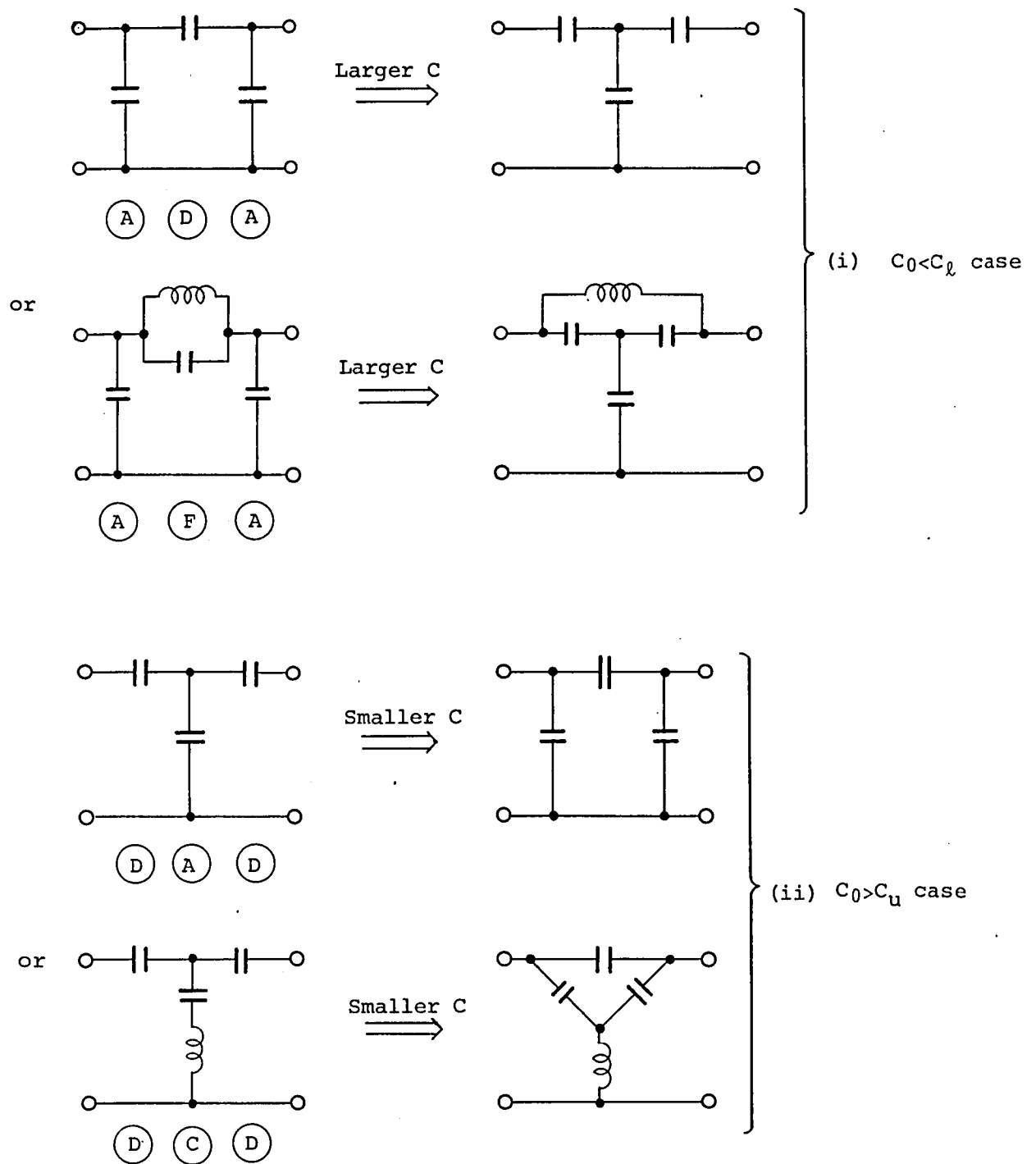


Fig. 4-6 T type-Π type circuit transformation.

- Example 1 --- 18th degree, $n_0 = 1$, $n_\infty = 3$, $C_\ell = 5\text{pF}$,
 $C_u = 10000\text{pF}$

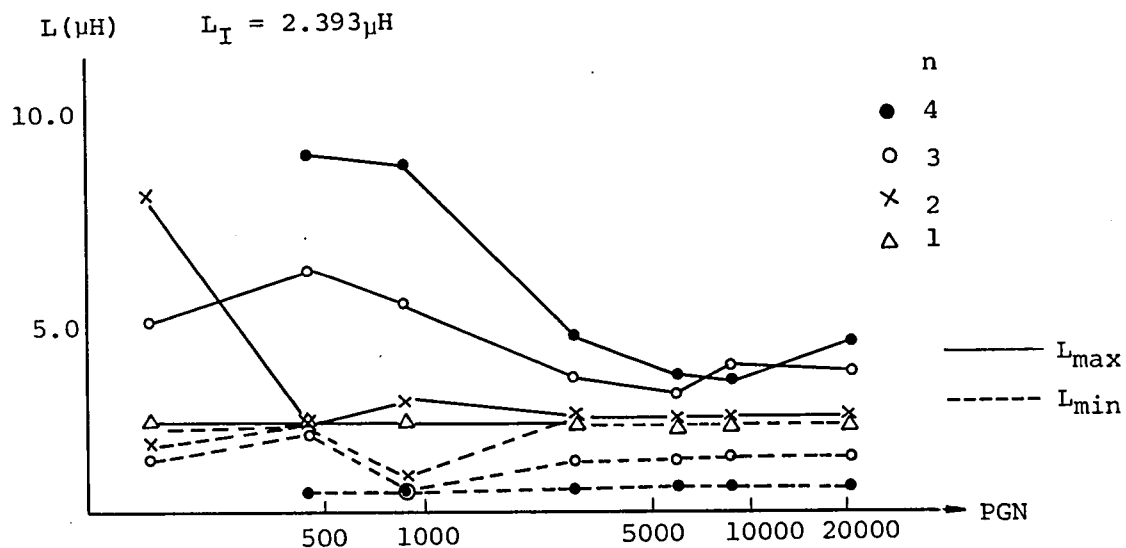


Fig. 4-7-a Example 1

- Example 2 --- 10th degree, $n_0 = 1$, $n_\infty = 3$, $C_\ell = 0$, $C_u = \infty$

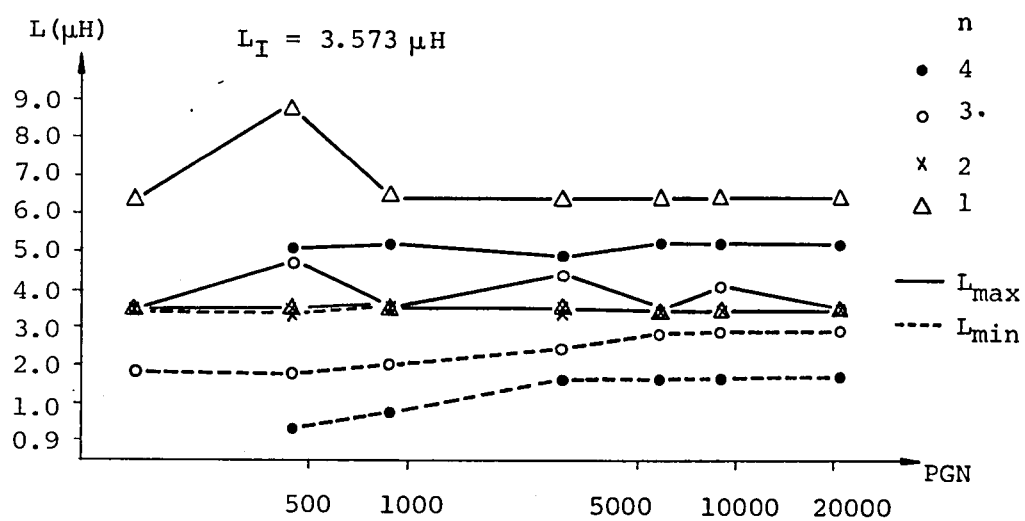


Fig. 4-7-b Example 2

- Example 3 --- 10th degree, $n_0 = 2$, $n_\infty = 2$, $C_\ell = 0$, $C_u = \infty$

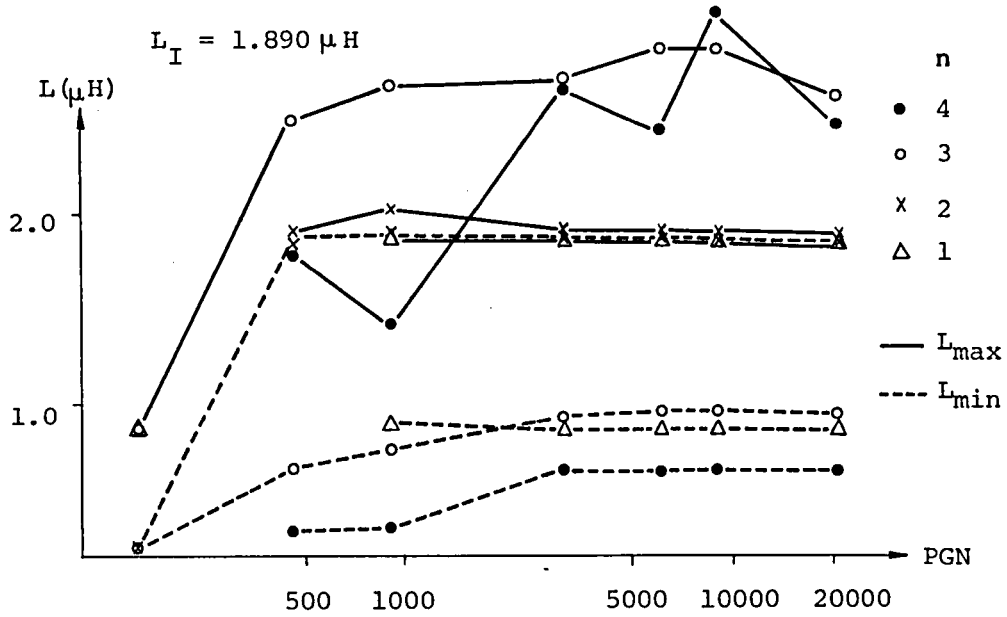


Fig. 4-7-c Example 3

- Example 4 --- 10th degree, $n_0 = 3$, $n_\infty = 1$, $C_\ell = 0$, $C_u = \infty$

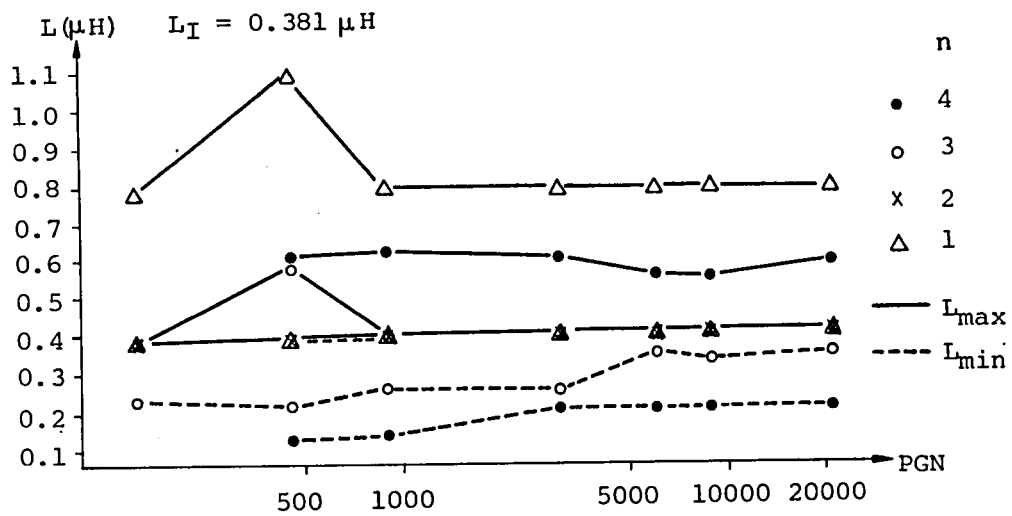


Fig. 4-7-d Example 4

Fig. 4-7 Relation between the pattern generation number and inductance value.

Fig. 4-7 says the followings.

(1) The algorithm successfully operates because L_{\max} and L_{\min} approach the ideal inductance L_I with the increase of PGN. In the figures, all the graphs give saturated curves for larger PGN.

(2) An appropriate number of ideal transformer is from one to three, regardless the filter degree. In cases of large number of ideal transformers, deviation from L_I is large because only few circuits can be subject to the 2-terminal impedance transformation. In another words, it shows that the element values improvement by the 2-terminal impedance transformation is considerably big. Therefore, in an actual design stage, the number of ideal transformer may be limited to three cases of one, two and three, so that the optimum equivalent circuit can be obtained for each case.

(3) As all the examples show that the optimum solution can be obtained by PGN of 1000 for one or two ideal transformers, 1000 may be sufficient for the practical number of PGN.

4-7 FLOW CHART OF COMPUTER PROGRAM

The flow chart of computer program is shown in Fig. 4-8.

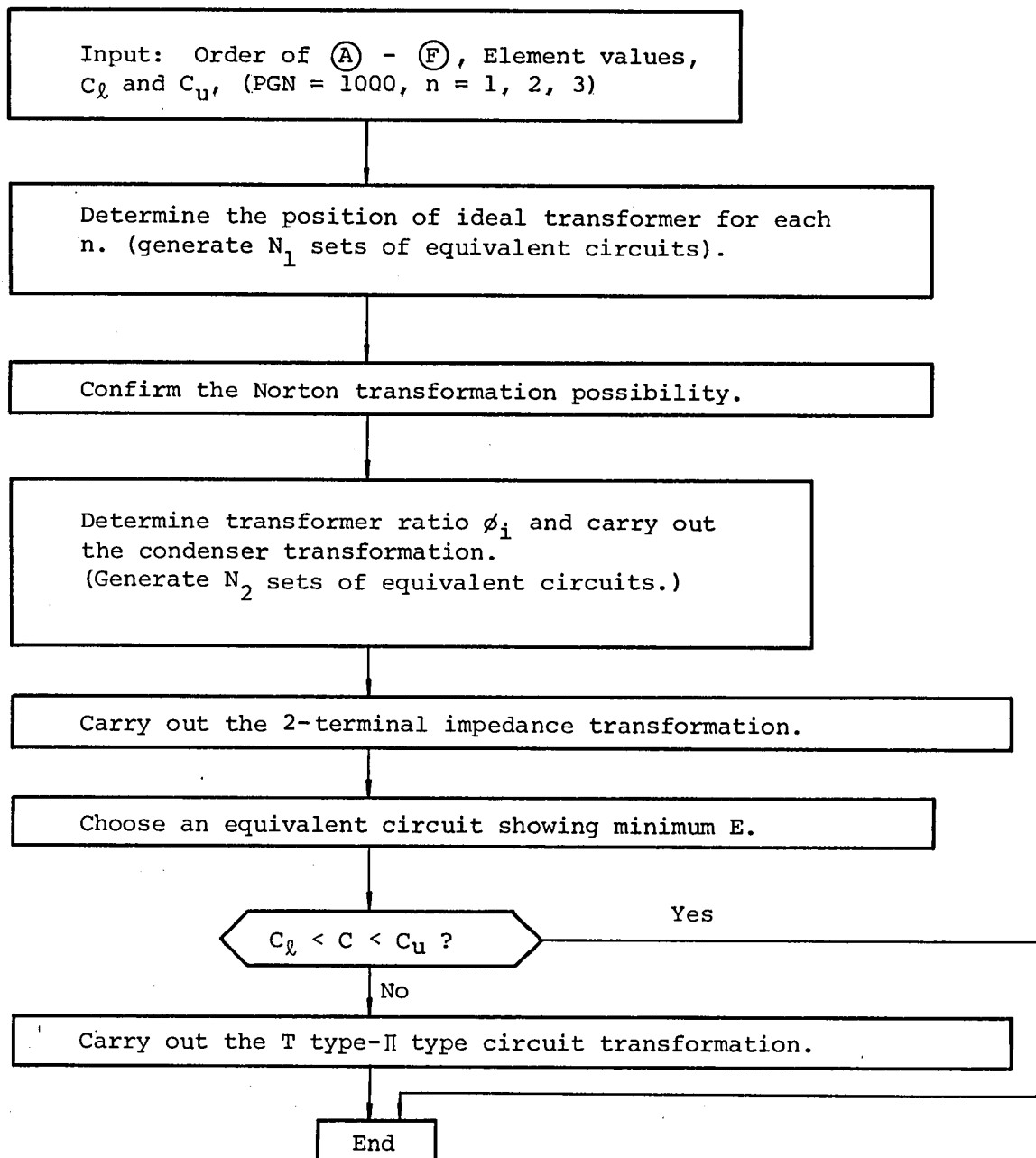
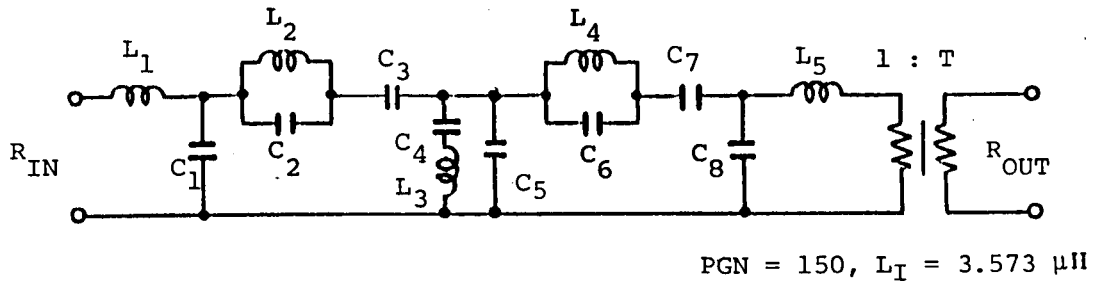


Fig. 4-8 Flow chart of computer program.

4-8 DESIGN EXAMPLE

For example 2 in section 4-6, the optimum circuit obtained by this method is shown in computer output together with an original circuit (see Fig. 4-9). Fig. 4-9-b shows a final circuit for $C_\ell=10\text{pF}$, $C_u=500\text{pF}$, and Fig. 4-9-c shows a final circuit for $C_\ell=0$, $C_u=\infty$. In the latter case, all the inductances have the same value as L_I . It takes 0.7 second for computation by IBM 370-168.



L_1	$3.57254703 \mu\text{H}$	C_1	50.4015857pF
L_2	6.87532165	C_2	19.5741617
L_3	8.88832710	C_3	9.80116509
L_4	3.25184737	C_4	47.4811856
L_5	3.81827977	C_5	44.2800796
T	1.03382015	C_6	45.7023308
		C_7	12.5726946
$R_{IN} = R_{OUT} = 75.0 \text{ohm}$		C_8	44.3530768

Fig. 4-9-a Original circuit

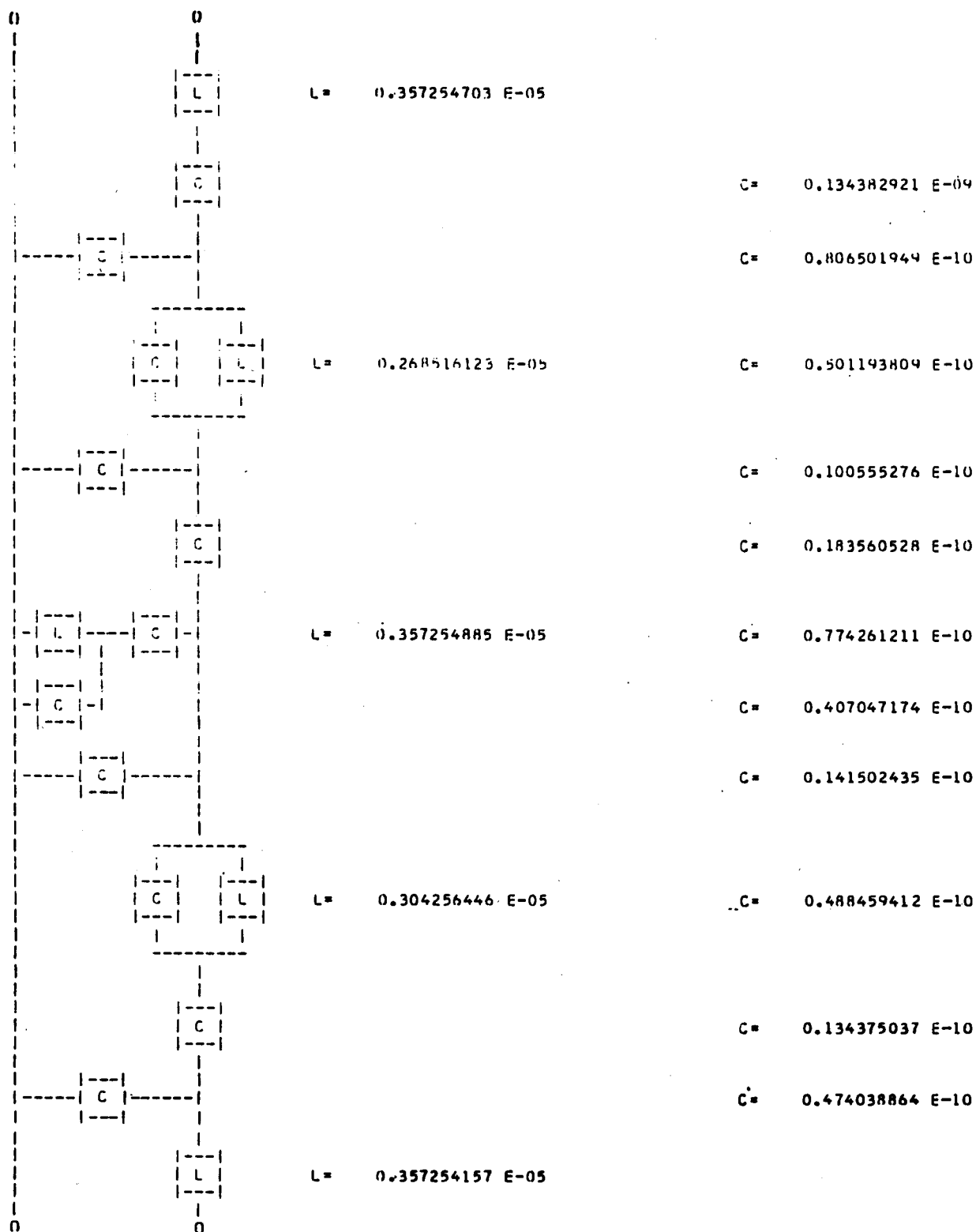


Fig. 4-9-b Final circuit for $C_L = 10\text{pF}$, $C_U = 500\text{pF}$

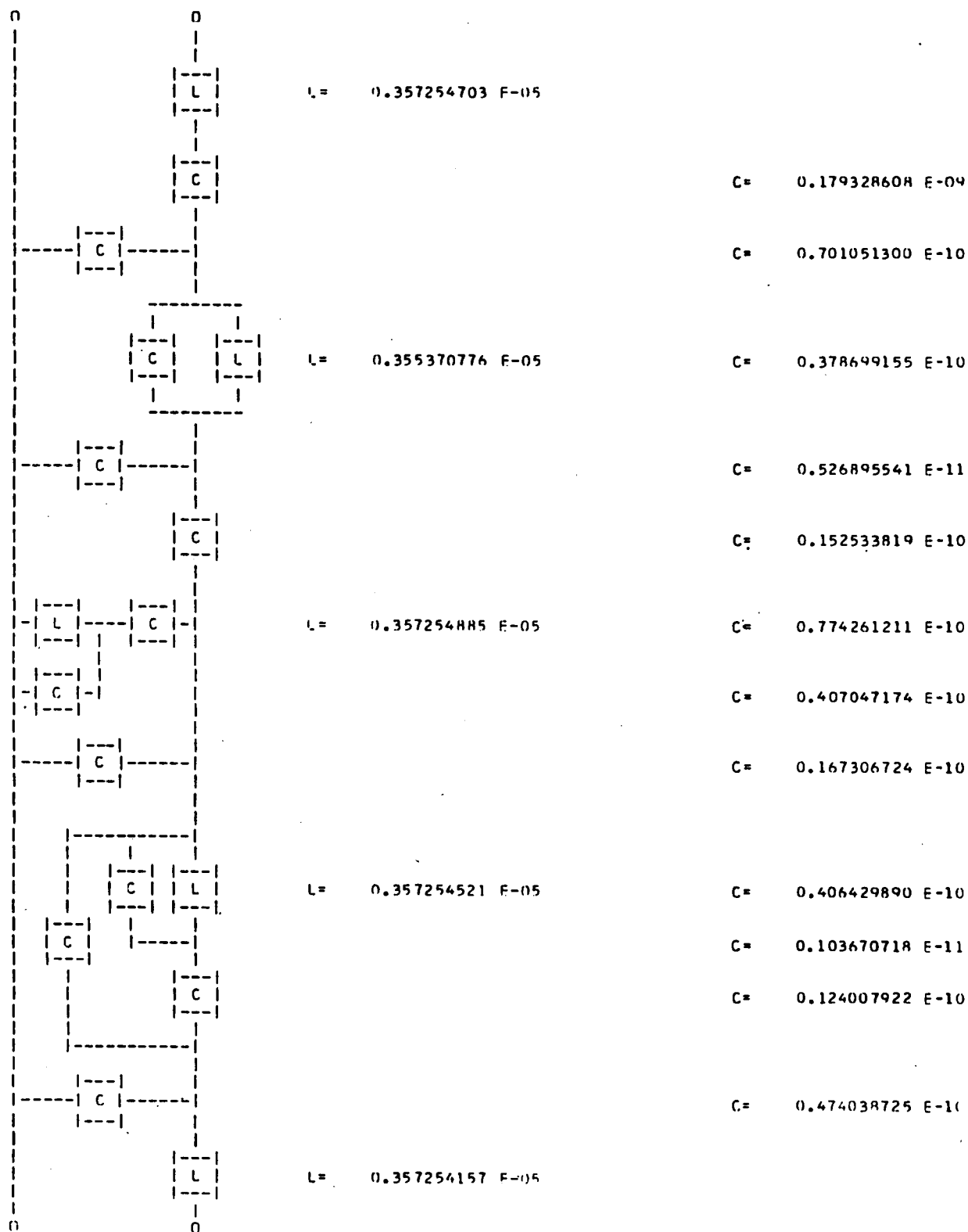


Fig. 4-9-c Final circuit for $C_\ell = 0$, $C_u = \infty$

Fig. 4-9 Circuit configuration and element values.

4-9 CONCLUSION

A CAD method of the Norton transformation which is one of the most important processes of band pass ladder filter design, was presented. So far, this process has required the engineer's hand operations. However, the present study has enabled the perfect design automation for this process, where the heuristic approaches using random number generator have been successfully used.

Some other formulations may be possible as an evaluation function. From many experiences, it is desired to have inductances of almost the same values in manufacturing stage. In this view point, we may have a following form as an evaluation function.

$$E = \frac{L_{\max}}{L_{\min}}$$

where E will approach to unity for the ideal case. ⁽²¹⁾

However, the above evaluation does not relate to the desired value L_I . Therefore, E in Eq.(4-4) is most preferable as an evaluation function.

CHAPTER 5. DESIGN OF FOUR-PORT DIRECTIONAL FILTER

5-1 INTRODUCTION

For submarine cable system, a single coaxial cable two-way transmission system is widely used from economical viewpoints. In this system, the transmitting frequency band is divided into two groups of lower and higher frequency bands, corresponding to the signal transmissions from east to west and from west to east. In this case, there are two types of submersible repeater construction.

One type of repeater is composed of two amplifiers as shown in Fig. 5-1, and two directional filters, which are both three-port network, are used to separate the lower and higher frequency bands.

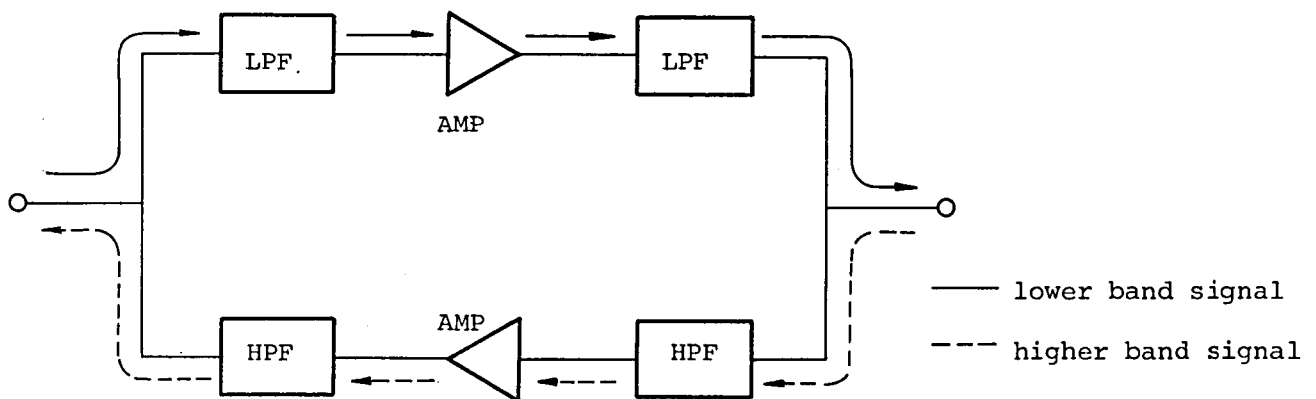


Fig. 5-1 A two amplifier configuration for repeater circuits.

Another type of repeater has a common amplifier configuration as shown in Fig. 5-2.

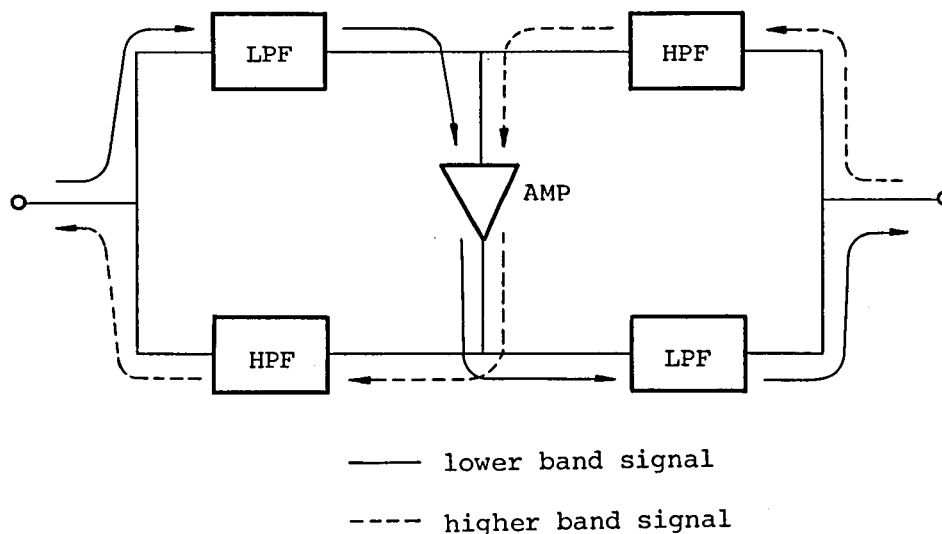


Fig. 5-2 A common amplifier configuration for repeater circuits.

The lower and higher band signal flows for both types are also shown in Fig. 5-1 and Fig. 5-2.

In order to realize a high reliability required for submersible repeater by reducing the number of parts per repeater, a common amplifier configuration is used. In this case, a four-port network is used for the directional filter as shown in Fig. 5-2. (23) (24)

C.A. Desoer⁽²⁵⁾ has presented a theory and a design method of a four-port directional filter used in this type of repeater. His method is based on the theory of four-port constant R network, but when it is applied to an actual design,

it becomes necessary to connect a one-port network (called compensation network) at each of the four ports for the following two reasons.⁽²⁶⁾ One is to compensate the susceptance which appears because of approximations in the design procedure. The other reason is to ensure sufficient loss between the two ports connected to the amplifier. Decrease of loss makes the repeater unstable because the directional filter constitutes a feedback path of the amplifier. As was pointed out in Desoer's paper, the loss can be made infinite in theory by inserting an ideal phase inversion transformer. In practice, however, the loss becomes considerably small because of inaccuracy of the element values and existence of parasitics.

In this chapter, a theory is proposed which enables to design a directional filter without use of compensation networks.⁽²⁷⁾⁽²⁸⁾⁽²⁹⁾ By this method, the same electrical performance can be achieved with less number of elements than by Desoer's method. Consequently, we can have filters more reliable and smaller in size. Directional filters of this new type are widely used in Japanese submarine cable systems.⁽³⁰⁾

5-2 ANALYSIS OF THE FOUR-PORT NETWORK

In the configuration of Fig. 5-3, the followings are assumed.

- (1) The two LPF are identical.
- (2) The two HPF are identical.

- (3) The four filters are reactive.
- (4) An ideal phase inversion transformer is inserted between ports 2 and 3.

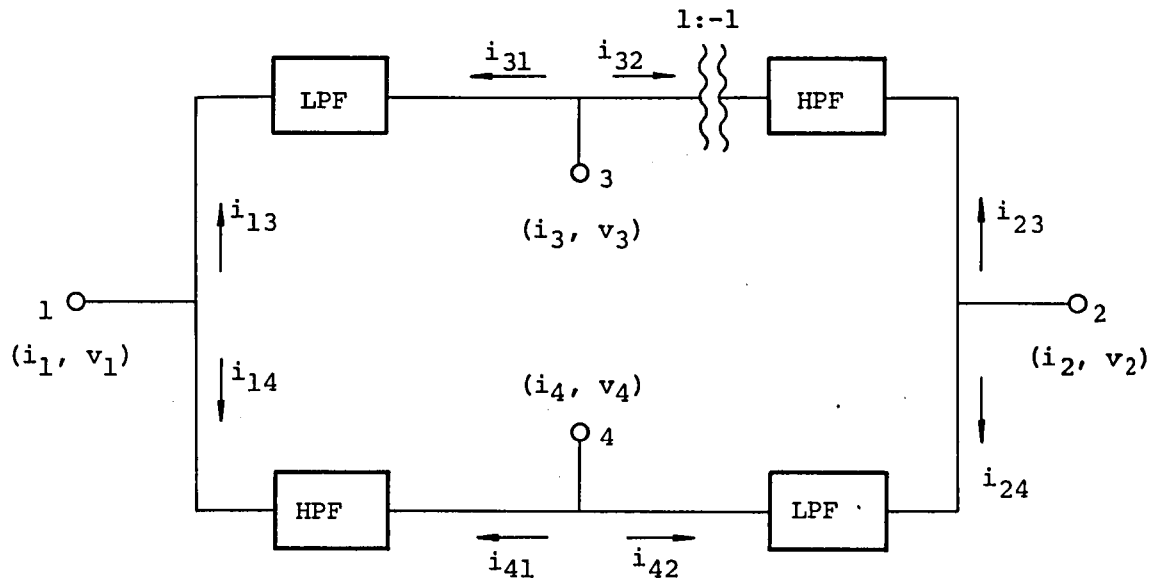


Fig. 5-3 The configuration of the four-port directional filter.

Let us define the admittance matrix of two-port network for each LPF or HPF as follows:

$$Y_L = \begin{bmatrix} Y_L^{(1)} & Y_{Lt} \\ Y_{Lt} & Y_L^{(2)} \end{bmatrix} \quad Y_H = \begin{bmatrix} Y_H^{(1)} & Y_{Ht} \\ Y_{Ht} & Y_H^{(2)} \end{bmatrix} .$$

[Lemma 5-1]

The admittance matrix Y of four-port network in Fig. 5-3, can be written as follows:

$$Y = \begin{bmatrix} y' & 0 & Y_{Lt} & Y_{Ht} \\ 0 & y' & -Y_{Ht} & Y_{Lt} \\ Y_{Lt} & -Y_{Ht} & y'' & 0 \\ Y_{Ht} & Y_{Lt} & 0 & y'' \end{bmatrix} \quad (5-1)$$

where $y' = y_L^{(1)} + y_H^{(1)}$ and $y'' = y_L^{(2)} + y_H^{(2)}$.

(Proof) Let us define the current and voltage at each port as (i_j, v_j) ($j=1, 2, 3, 4$), and the current i_{jk} from port j to port k ($j \neq k, k = 1, 2, 3, 4$).

From the Kirchhoff's current law at each port, we have

$$i_1 = i_{13} + i_{14}, \quad i_2 = i_{23} + i_{24}, \quad i_3 = i_{31} + i_{32},$$

and $i_4 = i_{41} + i_{42}.$

Each filter has a relation,

$$\begin{bmatrix} i_{13} \\ i_{31} \end{bmatrix} = \begin{bmatrix} Y_L^{(1)} & Y_{Lt} \\ Y_{Lt} & Y_L^{(2)} \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \end{bmatrix}, \quad \begin{bmatrix} i_{14} \\ i_{41} \end{bmatrix} = \begin{bmatrix} Y_H^{(1)} & Y_{Ht} \\ Y_{Ht} & Y_H^{(2)} \end{bmatrix} \begin{bmatrix} v_1 \\ v_4 \end{bmatrix}$$

$$\begin{bmatrix} i_{24} \\ i_{42} \end{bmatrix} = \begin{bmatrix} Y_L^{(1)} & Y_{Lt} \\ Y_{Lt} & Y_L^{(2)} \end{bmatrix} \begin{bmatrix} v_2 \\ v_4 \end{bmatrix}, \quad \begin{bmatrix} i_{23} \\ -i_{32} \end{bmatrix} = \begin{bmatrix} Y_H^{(1)} & Y_{Ht} \\ Y_{Ht} & Y_H^{(2)} \end{bmatrix} \begin{bmatrix} v_2 \\ -v_3 \end{bmatrix}.$$

From the above relations, we have

$$i_1 = (y_L^{(1)} + y_H^{(1)}) v_1 + y_{Lt} v_3 + y_{Ht} v_4$$

$$i_2 = (y_L^{(1)} + y_H^{(1)}) v_2 - y_{Ht} v_3 + y_{Lt} v_4$$

$$i_3 = y_{Lt} v_1 - y_{Ht} v_2 + (y_L^{(2)} + y_H^{(2)}) v_3$$

$$i_4 = y_{Ht} v_1 + y_{Lt} v_2 + (y_L^{(2)} + y_H^{(2)}) v_4$$

and, we obtain Eq.(5-1).

Q.E.D.

From the relation $S = (\Pi - Y)(\Pi + Y)^{-1}$ where S and Π express scattering matrix and unit matrix respectively, we have

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} = \begin{bmatrix} \rho' & 0 & t_L & t_H \\ 0 & \rho' & -t_H & t_L \\ t_L & -t_H & \rho'' & 0 \\ t_H & t_L & 0 & \rho'' \end{bmatrix}$$

where

$$\rho' = \frac{1}{\Delta} [(1-y')(1+y'') + (y_{Ht}^2 + y_{Lt}^2)] \quad (5-2-1)$$

$$\rho'' = \frac{1}{\Delta} [(1+y')(1-y'') + (y_{Ht}^2 + y_{Lt}^2)] \quad (5-2-2)$$

$$y' = y_H^{(1)} + y_L^{(1)}, \quad y'' = y_H^{(2)} + y_L^{(2)} \quad (5-2-3)$$

$$t_H = \frac{-2y_{Ht}}{\Delta} \quad (5-2-4)$$

$$t_L = \frac{-2y_{Lt}}{\Delta} \quad (5-2-5)$$

and

$$\Delta = (1 + y')(1 + y'') - (y_{Ht}^2 + y_{Lt}^2). \quad (5-2-6)$$

Here, we assume $\rho' = \rho'' = \rho$, that is, the reflection coefficient at each port, is equal. Therefore, we have $y' = y'' = y$.

[Lemma 5-2]

$$\text{If } y' = y'' = y, \text{ then } y_L^{(1)} = y_L^{(2)} = y_L \text{ and} \\ y_H^{(1)} = y_H^{(2)} = y_H.$$

(Proof) Let us define p_v , and p_μ as the poles of admittance matrix in LPF and HPF respectively. We can rewrite each admittance matrix of LPF and HPF in a form of common denominator.

$$y_L = \frac{1}{\prod_{v=1} (p - p_v)} \begin{bmatrix} f_L^{(1)}(p) & f_{Lt}(p) \\ f_{Lt}(p) & f_L^{(2)}(p) \end{bmatrix}$$

and

$$y_H = \frac{1}{\prod_{\mu=1} (p - p_\mu)} \begin{bmatrix} f_H^{(1)}(p) & f_{Ht}(p) \\ f_{Ht}(p) & f_H^{(2)}(p) \end{bmatrix}$$

where $f_L^{(1)}(p)$, $f_L^{(2)}(p)$, $f_{Lt}(p)$, $f_H^{(1)}(p)$, $f_H^{(2)}(p)$ and $f_{Ht}(p)$ express the numerator of $y_L^{(1)}$, $y_L^{(2)}$, y_{Lt} , $y_H^{(1)}$, $y_H^{(2)}$ and y_{Ht} respectively. Since, in general, the poles of admittance matrix in LPF and HPF are distinct each other, we have $p_v \neq p_\mu$.

From the relation $y_L^{(1)} - y_L^{(2)} = y_H^{(2)} - y_H^{(1)}$, we have

$$\frac{f_L^{(1)}(p)}{\prod_{v=1} (p-p_v)} - \frac{f_L^{(2)}(p)}{\prod_{v=1} (p-p_v)} = \frac{f_H^{(2)}(p)}{\prod_{\mu=1} (p-p_\mu)} - \frac{f_H^{(1)}(p)}{\prod_{\mu=1} (p-p_\mu)}$$

then,

$$\prod_{\mu=1} (p-p_\mu) \cdot (f_L^{(1)}(p) - f_L^{(2)}(p)) = \prod_{v=1} (p-p_v) \cdot (f_H^{(2)}(p) - f_H^{(1)}(p)).$$

When we consider the vicinity of $p = p_\mu$, the above equation approaches zero. Since $p_\mu \neq p_v$, we have $f_H^{(2)}(p) = f_H^{(1)}(p)$. Through the same discussion, we also have $f_L^{(2)}(p) = f_L^{(1)}(p)$. Therefore, we obtain $y_H^{(1)} = y_H^{(2)}$ and $y_L^{(1)} = y_L^{(2)}$. Q.E.D.

It means that each filter becomes symmetric. Then we have a following theorem.

[Theorem 7-1]

The scattering matrix of four-port network can be rewritten as,

$$S = \begin{bmatrix} \rho & 0 & \tau_L & \tau_H \\ 0 & \rho & -\tau_H & \tau_L \\ \tau_L & -\tau_H & \rho & 0 \\ \tau_H & \tau_L & 0 & \rho \end{bmatrix}$$

where

$$\rho = S_{11} = \frac{1-y_0}{1+y_0} \quad (5-3-1)$$

$$\tau_L = S_{13} = \frac{2}{1+y_0} \left(\frac{-y_{Lt}}{1+y} \right) \quad (5-3-2)$$

$$\tau_H = S_{14} = \frac{2}{1+y_0} \left(\frac{-y_{Ht}}{1+y} \right) \quad (5-3-3)$$

$$y_0 = y - \frac{y_{Ht}^2 + y_{Lt}^2}{1+y} \quad (5-3-4)$$

and

$$y = y_L + y_H \quad . \quad (5-3-5)$$

y_0 expresses the input admittance at each port.

(Proof) Eq.(5-3-1) is the definition of y_0 itself.

From Eq.(5-2-1) or (5-2-2) and Eq.(5-2-6),

$$\rho = \frac{1-y^2 + (y_{Ht}^2 + y_{Lt}^2)}{(1+y)^2 - (y_{Ht}^2 + y_{Lt}^2)} \quad .$$

Since $y_0 = \frac{1-\rho}{1+\rho}$,

$$\begin{aligned}
y_0 &= \frac{1 - \frac{1 - y^2 + y_{Ht}^2 + y_{Lt}^2}{(1+y)^2 - (y_{Ht}^2 + y_{Lt}^2)}}{1 + \frac{1 - y^2 + y_{Ht}^2 + y_{Lt}^2}{(1+y)^2 - (y_{Ht}^2 + y_{Lt}^2)}} \\
&= \frac{y + y^2 - (y_{Ht}^2 + y_{Lt}^2)}{(1 + y)} \\
&= y - \frac{y_{Ht}^2 + y_{Lt}^2}{1 + y} \quad . \quad (\text{Eq. (5-3-4)})
\end{aligned}$$

This relation can be written as,

$$y - y_0 = \frac{y_{Ht}^2 + y_{Lt}^2}{1 + y} \quad .$$

Substituting the above relation into Eq.(5-2-4),

$$\begin{aligned}
t_H &= \frac{-2y_{Ht}}{(1+y)^2 - (y_{Ht}^2 + y_{Lt}^2)} \\
&= \frac{-2y_{Ht}}{(1+y)^2 - (y - y_0)(1+y)} \\
&= \frac{-2y_{Ht}}{(1+y)(1+y_0)} = \tau_H \quad . \quad (\text{Eq. (5-3-3)})
\end{aligned}$$

Q.E.D.

Since the network is reactive, using Eq.(5-3-1)

$$\begin{aligned} S_{13}\tilde{S}_{13} + S_{14}\tilde{S}_{14} &= 1 - S_{11}\tilde{S}_{11} \\ &= \frac{2}{1+y_0} \cdot \frac{2}{1+\tilde{y}_0} \cdot \text{Ev } y_0, \end{aligned} \quad (5-4)$$

where \sim denotes the substitution of p by $-p$, and $\text{Ev } y_0$ expresses even part of y_0 .

Let $G(p)$ be even part of y_0 , then reminding that y_H and y_L are odd rational functions of p (for example, see Eq.(3-2) in Chapter 3) and then, since $\tilde{y} = -y$, we have

$$\begin{aligned} G(p) &= \text{Ev } y_0 = \frac{y_0 + \tilde{y}_0}{2} \\ &= \frac{1}{2} (y + \tilde{y} - (y_{Ht}^2 + y_{Lt}^2) (\frac{1}{1+y} + \frac{1}{1+\tilde{y}})) \\ &= \frac{y_{Ht}^2 + y_{Lt}^2}{y^2 - 1} \end{aligned} \quad (5-5)$$

or

$$y_{Ht}^2 + y_{Lt}^2 = (y^2 - 1) G(p). \quad (5-6)$$

The above relations (5-5) and (5-6) are important for our problem.

Let Ω_H , Ω_L and Ω_T denote the higher passband, the lower passband and the transition band respectively. Combining (5-3) and (5-6), we have

$$\frac{1}{|S_{14}|^2} = \left(\frac{1+y_0}{2}\right) \left(\frac{1+\hat{y}_0}{2}\right) \cdot \frac{y_{Ht}^2 + y_{Lt}^2}{y_{Ht}^2 G(p)} \quad (5-7)$$

$$\frac{1}{|S_{13}|^2} = \left(\frac{1+y_0}{2}\right) \left(\frac{1+\hat{y}_0}{2}\right) \cdot \frac{y_{Ht}^2 + y_{Lt}^2}{y_{Lt}^2 G(p)} .$$

Since $G(p)$ is close to unity in the passband, (5-7) has the following approximations:

$$\frac{1}{|S_{14}|^2} \approx 1 + \frac{y_{Lt}^2}{y_{Ht}^2} \quad \text{for } \Omega \in \Omega_L \quad (5-8)$$

$$\frac{1}{|S_{13}|^2} \approx 1 + \frac{y_{Ht}^2}{y_{Lt}^2} \quad \text{for } \Omega \in \Omega_H .$$

Since, in Ω_L , that is, in the stopband of HPF, the attenuation loss must be extremely larger than that of LPF and, in Ω_H , that is, in the stopband of LPF, the attenuation loss must be extremely larger than that of HPF, we have the relations,

$$|y_{Lt}|^2 \gg |y_{Ht}|^2 \quad \text{for } \Omega \in \Omega_L \quad (5-9)$$

$$|y_{Ht}|^2 \gg |y_{Lt}|^2 \quad \text{for } \Omega \in \Omega_H$$

and

$$y_{Lt}^2 + y_{Ht}^2 \approx y_{Lt}^2 \quad \text{for } \Omega \in \Omega_L \quad (5-10)$$

$$y_{Lt}^2 + y_{Ht}^2 \approx y_{Ht}^2 \quad \text{for } \Omega \in \Omega_H .$$

From Eq. (5-9), Eq. (5-8) becomes

$$\frac{1}{|S_{14}|^2} \approx \frac{y_{Lt}^2}{y_{Ht}^2} \quad \text{for } \Omega \in \Omega_L \quad (5-11)$$

$$\frac{1}{|S_{13}|^2} \approx \frac{y_{Ht}^2}{y_{Lt}^2} \quad \text{for } \Omega \in \Omega_H .$$

Let us define filter characteristic functions ψ_H and ψ_L for HPF and LPF as,

$$\psi_H = \frac{-1}{y_{Ht}} \quad \text{and} \quad \psi_L = \frac{-1}{y_{Lt}} .$$

Using Eq. (5-11), the attenuation loss A_H and A_L for HPF and LPF can be approximately written as

$$A_H = 10 \log_{10} \left| \frac{1}{S_{14}} \right|^2 \approx 10 \log_{10} |\psi_H|^2 - 10 \log_{10} |\psi_L|^2$$

$$\geq A_{OH} \quad \text{for } \Omega \in \Omega_L$$

$$A_L = 10 \log_{10} \left| \frac{1}{S_{13}} \right|^2 \approx 10 \log_{10} |\psi_L|^2 - 10 \log_{10} |\psi_H|^2$$

$$\geq A_{OL} \quad \text{for } \Omega \in \Omega_H \quad (5-12)$$

where A_{OH} and A_{OL} express the given attenuation loss specifications for HPF and LPF respectively. Here, Eq. (5-6) is replaced by

$$\psi_H^{-2} + \psi_L^{-2} = (y^2 - 1)G(p) . \quad (5-13)$$

Our design problem is to obtain ψ_L and ψ_H solving Eq.(5-13) under the constraint of Eq.(5-12).

Let us consider the requirements imposed on the above functions $S_{11}(G(p))$, S_{13} , S_{14} and S_{34} , etc. For the filters, the loop attenuation between ports 3 and 4 must be sufficiently large to decrease the passband ripple of a repeater. And also, in the transition band, the filters must, at least, have larger attenuation than the amplifier's gain.

Now, the sensitivity of the operating transfer function between the ports 3 and 4 with respect to the terminating resistor r_1 at the port 1 is

$$\left| \frac{\partial S_{34}}{\partial r_1} \right|^2 \propto |S_{13}|^2 \cdot |S_{14}|^2 \text{ on the } j\Omega \text{ axis.}$$

In order to ensure the stability of a repeater, the sensitivity must be small.

Since the high return loss is required in Ω_H and Ω_L , $|S_{11}|^2 \approx 0$ for $\Omega \in \Omega_H \cup \Omega_L$. In Ω_H , $|S_{13}|^2$ is very small in comparison with a magnitude of $|S_{14}|^2$ and, also in Ω_L , $|S_{14}|^2$ is very small in comparison with a magnitude of $|S_{13}|^2$. Accordingly, the sensitivity which is proportional to the product of $|S_{13}|^2$ and $|S_{14}|^2$, becomes small.

In Ω_T , however, the magnitude of $|S_{13}|^2$ and $|S_{14}|^2$ are comparable, hence, in order to keep the sensitivity small, the sum $|S_{13}|^2 + |S_{14}|^2$ must be small.

Then, the requirements for the filters are summarized as follows:

- (1) $G(p)$ must be close to unity so that the filters have high return loss in the passbands. (Eq.(5-5)).
- (2) S_{13} and S_{14} (S_{23} and S_{24}) must be so determined that the filters have small sensitivity of the operating transfer function S_{34} between the ports 3 and 4. This is reflected to $G(p)$ by the relation (5-5).
- (3) S_{13} and S_{14} (S_{23} and S_{24}), also, must be so determined that the filters have enough attenuation in all bands.

5-3 NETWORK REALIZATION

In Eq.(5-6), we can find that the sum $y_{Ht}^2 + y_{Lt}^2$ must have maxima of value $-G(p)$ on the $j\Omega$ axis. From Eq.(5-10), y_{Ht}^2 , for $\Omega \in \Omega_H$, has maxima of value $-G(p)$, which means that $|\psi_H|^2$ oscillates between 0 and $1/G(p)$. The same for y_{Lt}^2 holds for $\Omega \in \Omega_L$. Desoer's theorem for the necessary and sufficient conditions of physical realizability is modified and stated as follows:

- (1) y_{Lt} and y_{Ht} must be odd rational functions of p , having simple poles located on the $j\Omega$ axis.
- (2) The $j\Omega$ poles y_{Lt} are distinct from those of y_{Ht} .
- (3) For $p = j\Omega$,

$$y_{Lt}^2 + y_{Ht}^2$$

must have maxima of value $-G(p)$.

The condition (3) is different from Desoer's theorem and is important for the problem. Under these conditions, we consider the physical realization.

Let the partial fraction expansions of y_{Lt} and y_{Ht} be, respectively,

$$y_{Lt} = \sum'_{-n} \frac{k_i}{p-j\omega_i} + \frac{k_0}{p} \quad (5-14)$$

$$y_{Ht} = \sum'_{-m} \frac{h_k}{p-jq_k} + h_{\infty}p ,$$

where the prime indicates that the terms for $n, m = 0$ are omitted and

$$\begin{aligned} \omega_i &= -\omega_{-i} & (i = 1, 2, \dots, n) \\ q_k &= -q_{-k} & (k = 1, 2, \dots, m) \\ \omega_i &\neq q_k & (i = 1, 2, \dots, n; k = 1, 2, \dots, m). \end{aligned}$$

Now, let us consider the relation (5-6). When Eq.(5-6) is expanded in Laurent series around the points $p = j\omega_i$, $p = jq_k$, $p = j0$ and $p = j\infty$, the dominant term of the left hand side must match the dominant term of the right hand side.

Therefore, y can be written as:

$$y = \sum'_{-n} \frac{k'_i}{p-j\omega_i} + \sum'_{-m} \frac{h'_k}{p-jq_k} + \frac{k'_0}{p} + h'_{\infty}p .$$

Then, the residue matching conditions are as follows:

$$k_i'^2 \cdot G(p) = k_i^2 \quad \text{at } p = j\omega_i, \quad k_0'^2 \cdot G(p) = k_0^2 \quad \text{at } p = j0$$

$$h_k'^2 \cdot G(p) = h_k^2 \quad \text{at } p = jq_k, \quad h_\infty'^2 \cdot G(p) = h_\infty^2 \quad \text{at } p = j\infty.$$

The following conditions must be added so that LPF and HPF are reactive.

$$k_i'^2 - k_i^2 \geq 0, \quad k_0'^2 - k_0^2 \geq 0$$

$$h_k'^2 - h_k^2 \geq 0, \quad h_\infty'^2 - h_\infty^2 \geq 0.$$

Consequently, we have

$$k_i'^2 (1-G(p)) \geq 0 \quad \text{at } p = j\omega_i, \quad k_0'^2 (1-G(p)) \geq 0 \quad \text{at } p=j0$$

$$h_k'^2 (1-G(p)) \geq 0 \quad \text{at } p = jq_k, \quad h_\infty'^2 (1-G(p)) \geq 0 \quad \text{at } p=j\infty.$$

Combining above conditions, we have

$$G(p) \leq 1 \quad \text{at } p = j\omega_i, \quad p = jq_k, \quad p = j0 \quad \text{and } p = j\infty.$$

The case $G(p) = 1$ at $p = j\omega_i, p = jq_k, p = j0$ and $p = j\infty$, satisfies the minimum susceptance condition. This requirement means that the zeros of ψ_L, ψ_H must exist at the point of $G(p) = 1$. So, we can have the reactance functions y_L and y_H by

$$Y_L = \sum_{-n}^{+n} \frac{|k_i|}{p-j\omega_i} + \frac{|k_0|}{p} \quad (5-15)$$

$$Y_H = \sum_{-m}^{+m} \frac{|h_k|}{p-jq_k} + |h_\infty|p$$

5-4 DESIGN PROCEDURE

The actual design procedure together with an example is shown as follows:

(1) Determine $G(p)$ so that it satisfies the requirements of the return loss in the passbands, at the same time, gives the small sensitivity in the transition band. In order to determine $G(p)$, we consider reference filters⁽³¹⁾ which satisfy the above requirements.

Let ρ_H and ρ_L be the reflection coefficients of the reference high pass and low pass filters respectively, and we obtain the following⁽³¹⁾:

$$\rho = \rho_H \cdot \rho_L$$

where

$$\rho = \frac{1-Y_0}{1+Y_0}, \quad \rho_H = \frac{1-Y_H}{1+Y_H} \quad \text{and} \quad \rho_L = \frac{1-Y_L}{1+Y_L}.$$

Y_H and Y_L express the admittances of the reference high pass and low pass filters.

From the relation $G(p) = Ev y_0$, we have

$$G(p) = Ev \frac{Y_H + Y_L}{1 + Y_H Y_L} .$$

Fig. 5-4 shows a given $G(p)$.

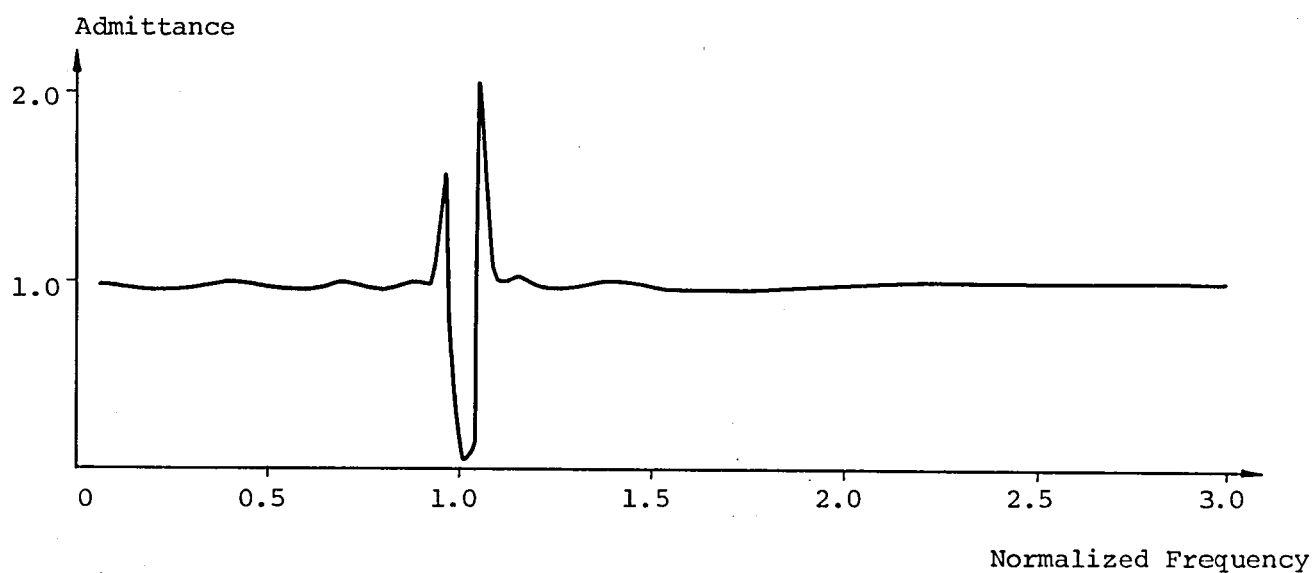


Fig. 5-4 The characteristics of given $G(p)$.

- (2) Find the poles of ψ_H and ψ_L from the requirements in the attenuation bands.
- (3) Modify the zeros of ψ_H and ψ_L so that $|\psi_H|^2$ and $|\psi_L|^2$ do not exceed $1/G(p)$ and are tangent to $1/G(p)$ in their respective band, by using the iterative approximation method⁽³²⁾. Fig. 5-5 shows $|\psi_H|^2$ and $|\psi_L|^2$.

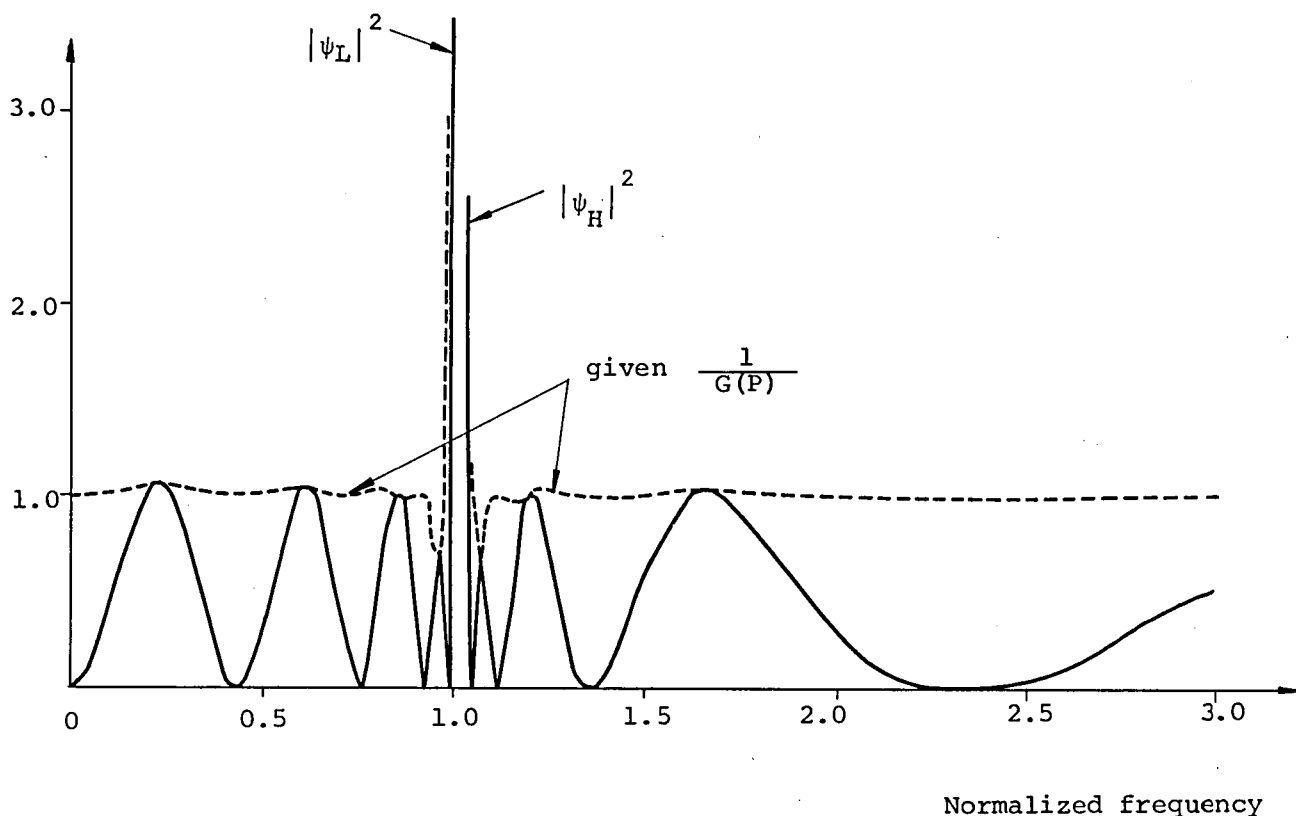


Fig. 5-5 $|\psi_H|^2$ and $|\psi_L|^2$ after the design procedure (3).

(4) Obtain y_H and y_L which satisfy the residue matching requirement (see Eq.(5-15)).

(5) Proceed the cascade synthesis of y_H and y_L .

The characteristics obtained through these procedures are shown in Figs. 5-6, 5-7, 5-8 and 5-9. Figs. 5-6 and 5-7 show the even part of the input admittance and the return loss at each port, respectively. Fig. 5-8 shows the insertion loss between ports 1 and 3, and between ports 1 and 4. Fig.

5-9 shows the loop loss between ports 3 and 4. Fig. 5-10 shows the network configuration and element values.

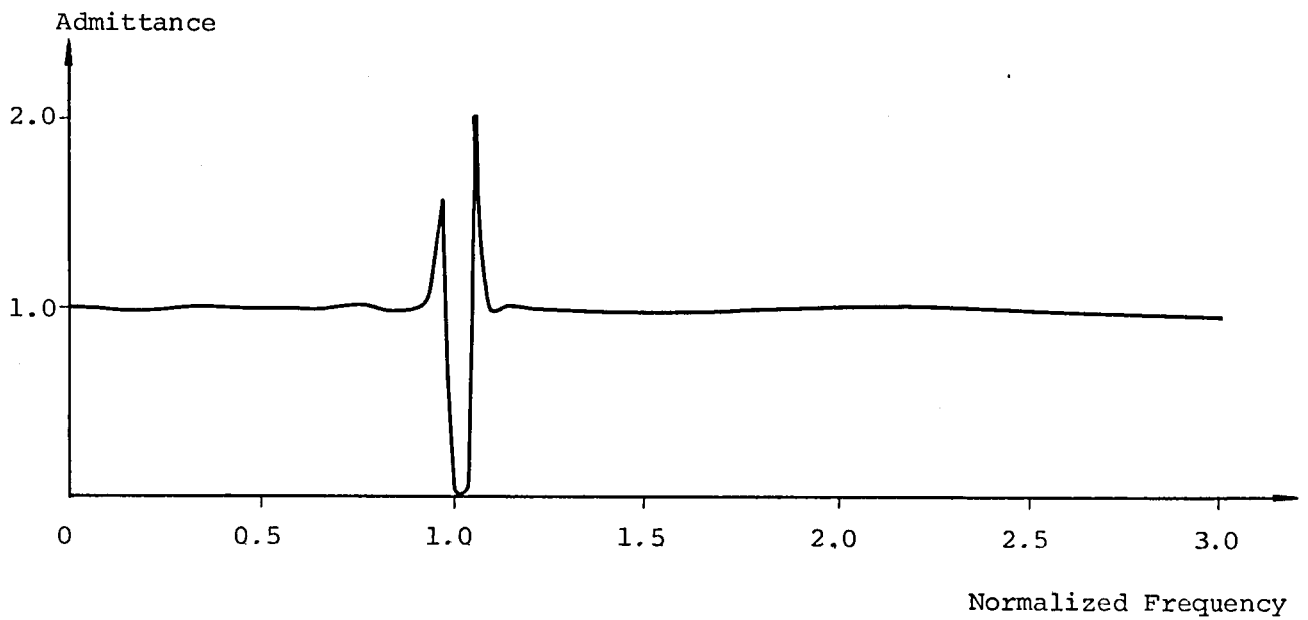


Fig. 5-6 Even part of the input admittance characteristics.

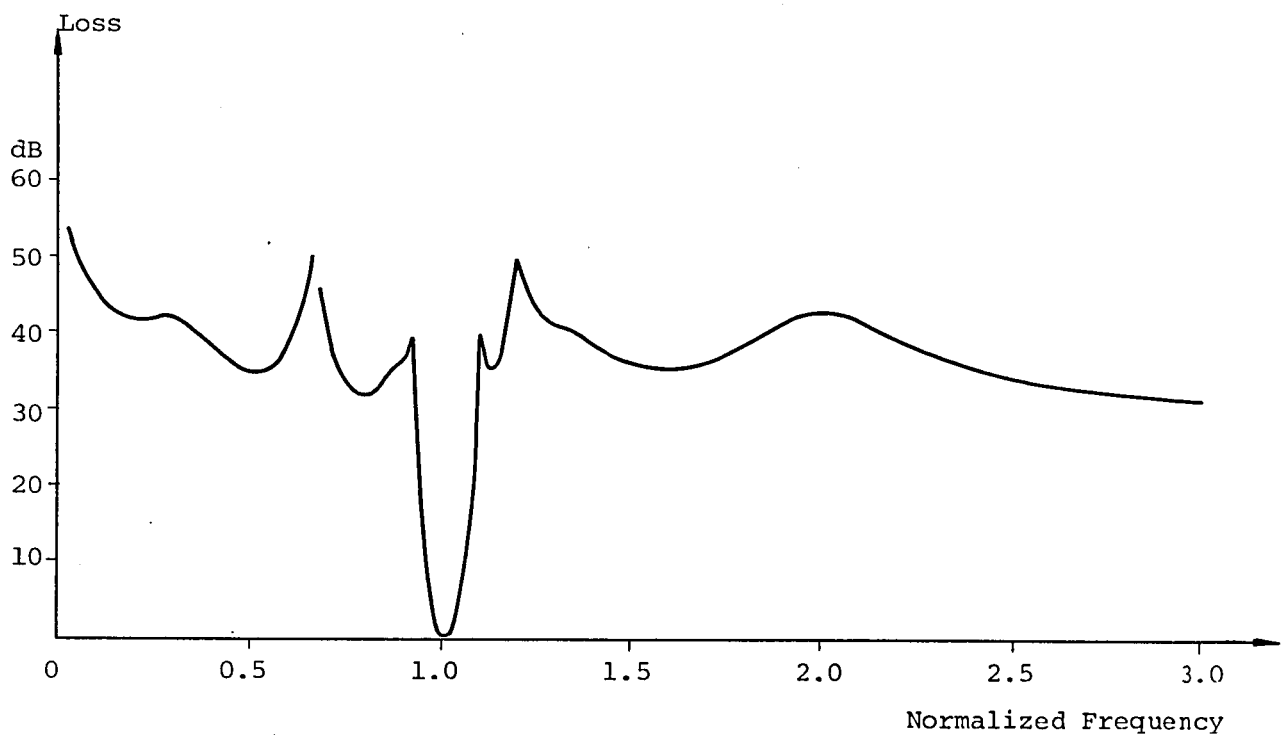


Fig. 5-7 Return loss characteristics.

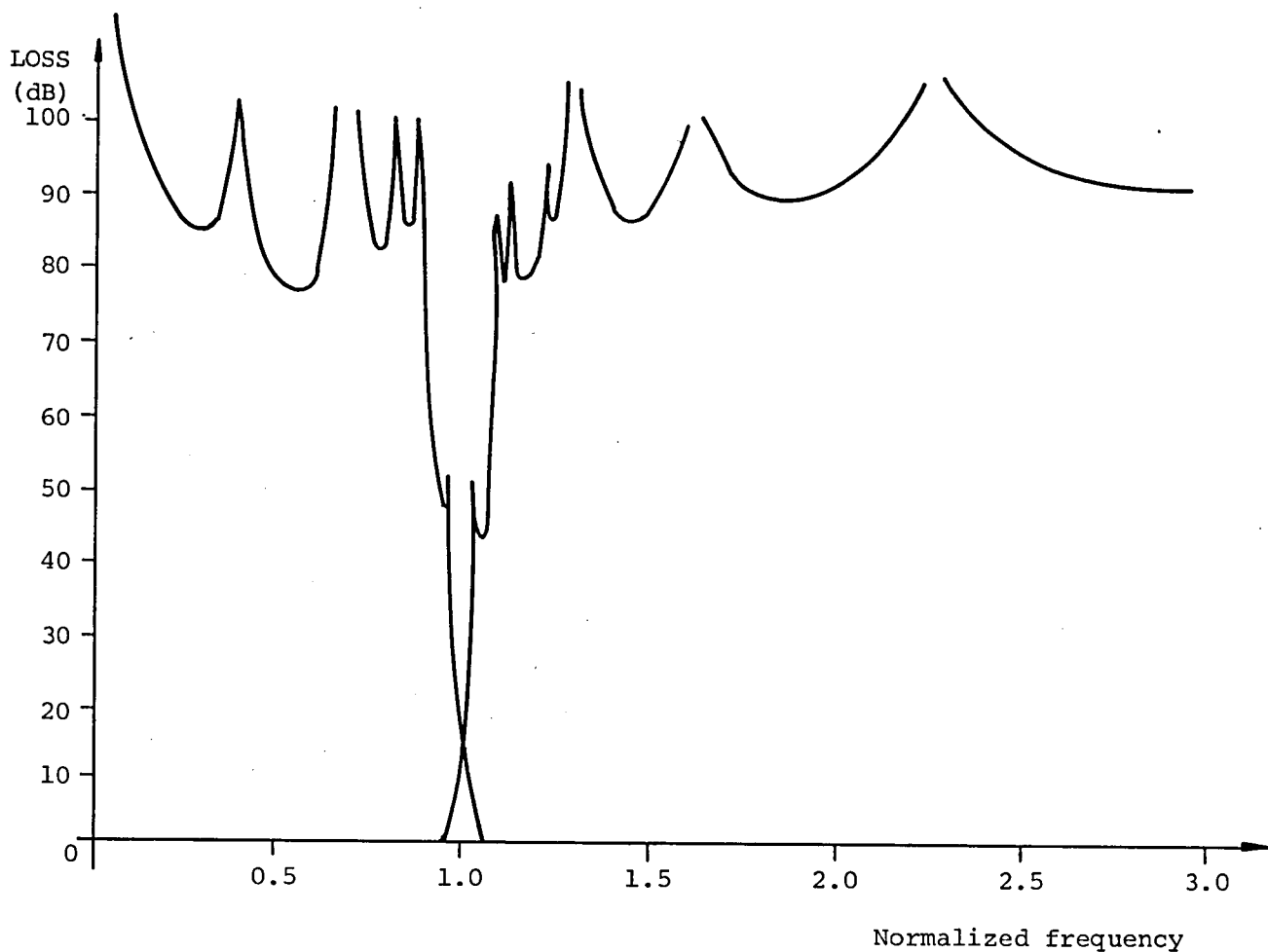


Fig. 5-8 Insertion loss characteristics between ports 1 and 3, and between ports 1 and 4.

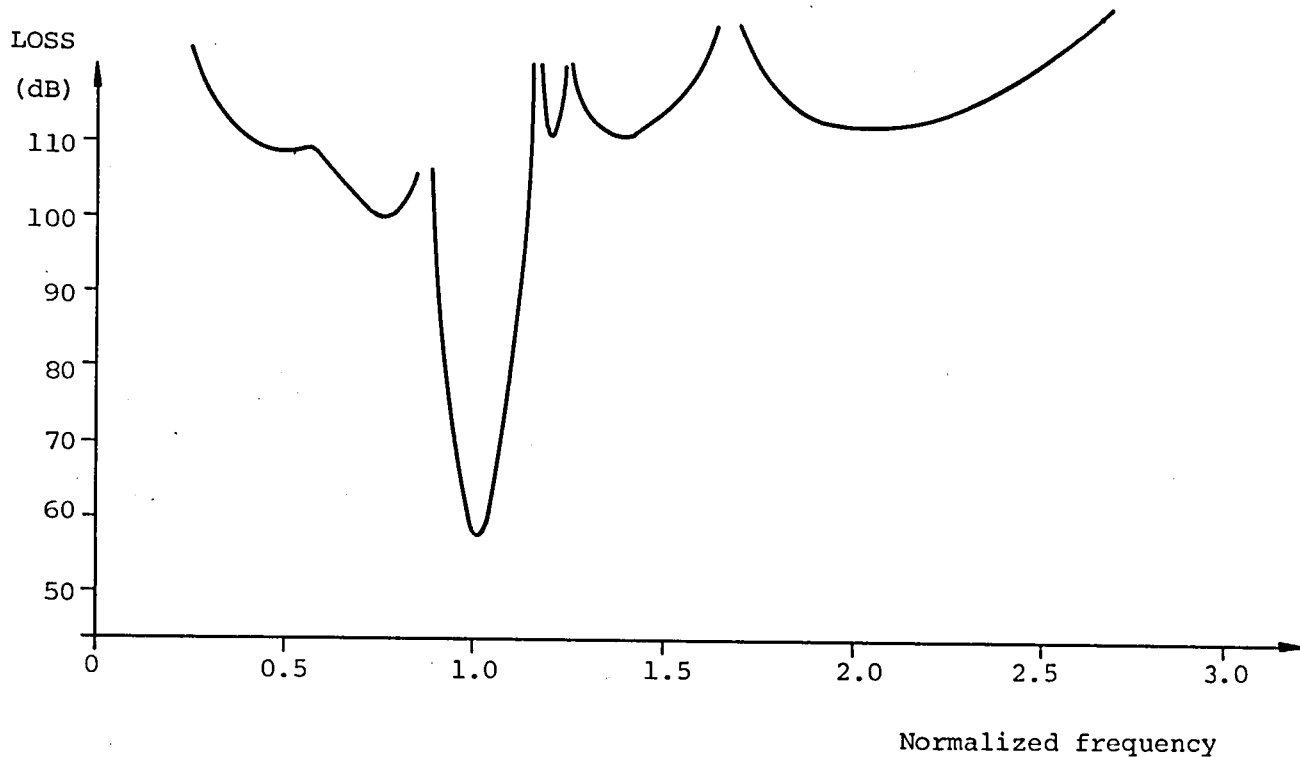


Fig. 5-9 Loop loss characteristics between ports 3 and 4.

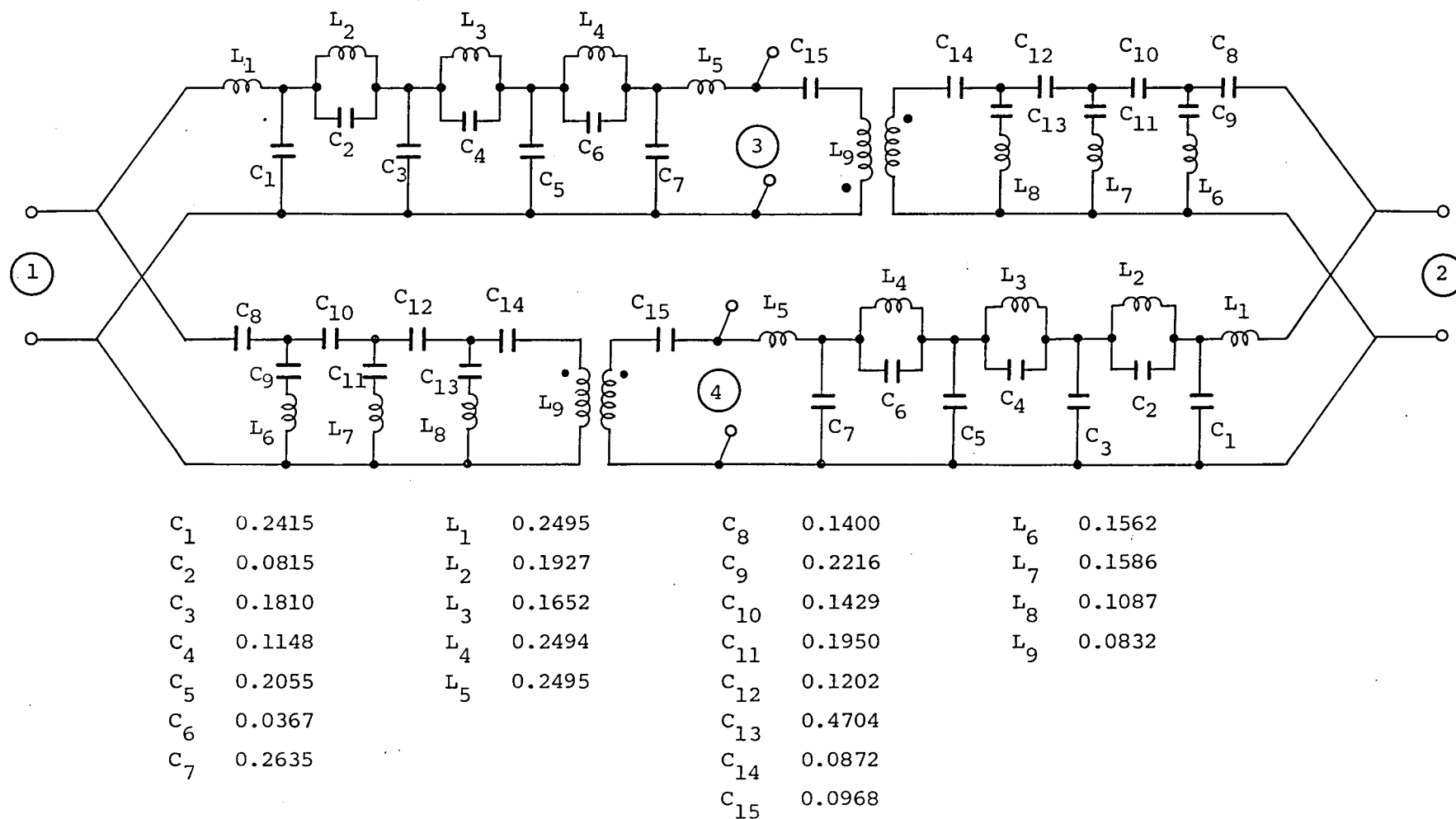


Fig. 5-10 Network configuration and element values.

In the actual hardware realization, adjustable air core inductors with high-Q characteristics are used in order to meet the critical requirements, such as low distortion,⁽³³⁾ small passband ripple and ensurance of sufficient loop loss. Naked golden mica capacitors are adopted because of their high-Q characteristics.⁽³⁴⁾ Each coil is shielded to prevent from harmful coupling. Volume of each separately shielded section is decided according to the sensitivity of inductor's Q in the network because inductor's Q is affected by its occupied space. This "honeycomb" shape of each section is shown in Fig. 5-11.

In addition, we realized a network in which an inductor is used as phase inversion transformer and filter element (see Fig. 5-10). For comparison with the newly developed filter, the network configuration of a directional filter designed by the converntional method is shown in Fig. 5-12. From Fig. 5-10 and Fig. 5-12, it is seen that eleven elements are reduced.

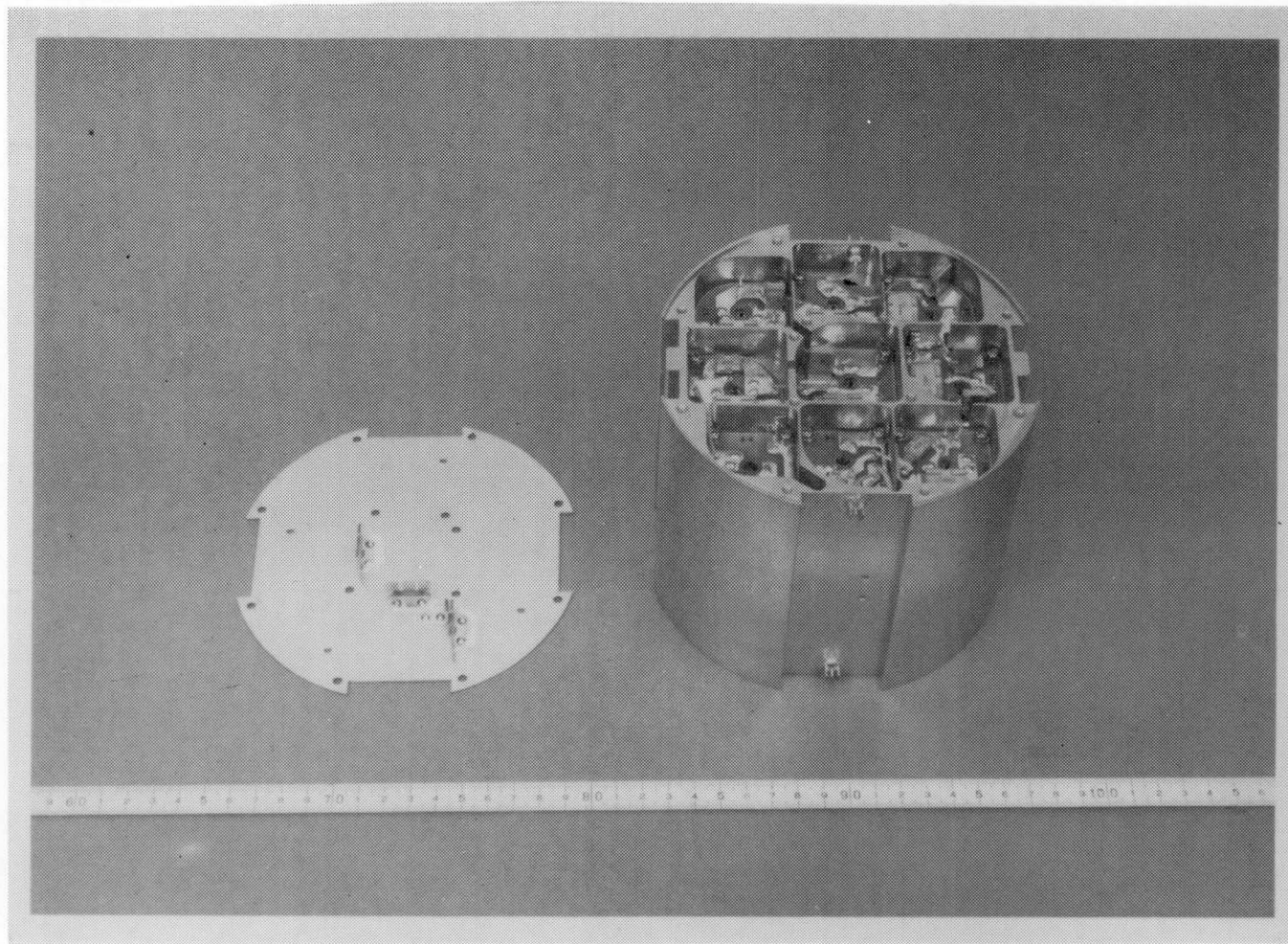


Fig. 5-11 Honeycomb construction of a four-port directional filter.

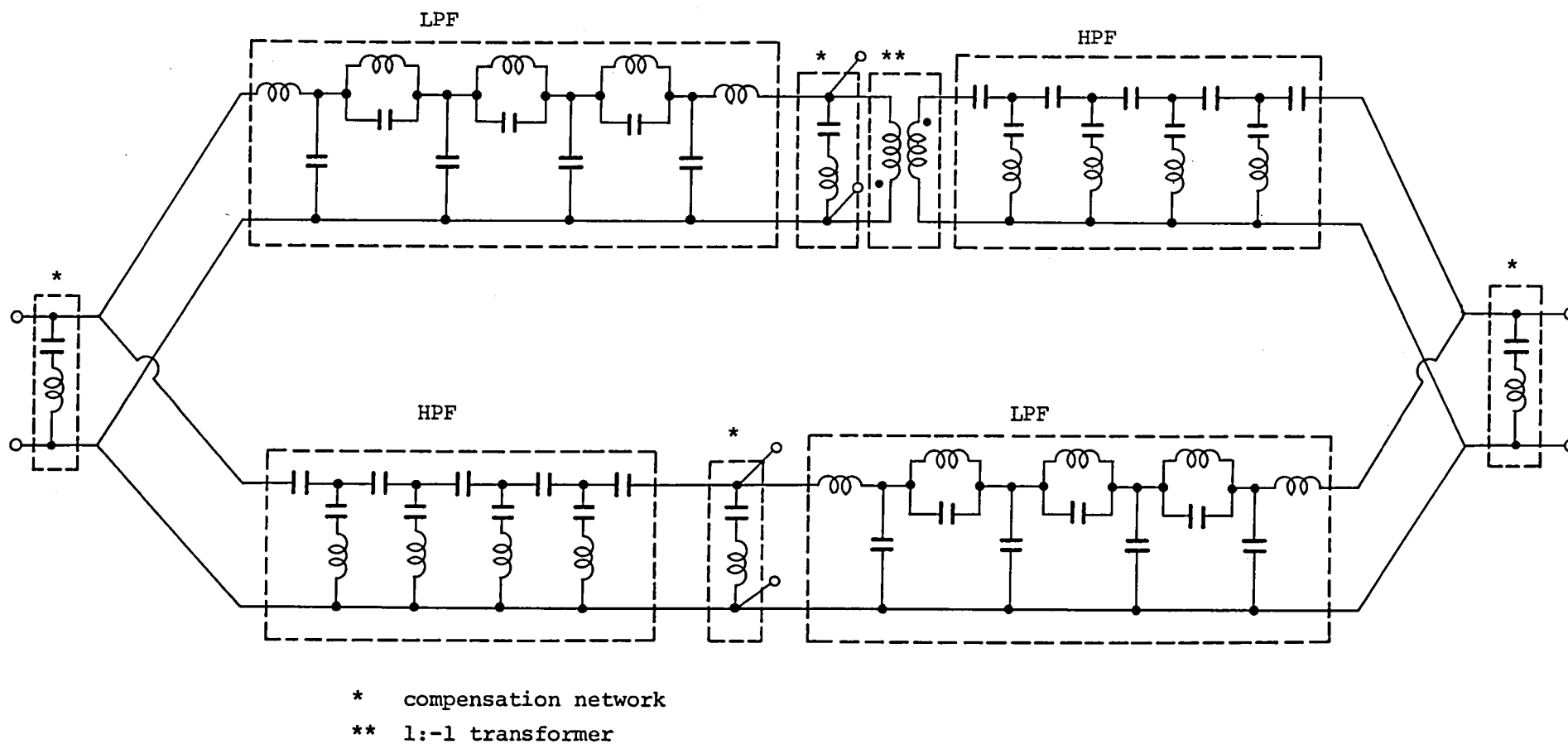


Fig. 5-12 Network configuration by the conventional method.

5-5 CONCLUSION

The design method of a four-port directional filter which is commonly used in a submarine cable system was described. By this method, we were able to have a filter without using compensation networks, with the same performances as a filter designed by the conventional method, which requires compensation networks at each port of directional filter. This contributes not only to reduce filter volume but also to realize higher reliability. This filter is widely used in submarine cable systems in Japan to improve the system economy and reliability.

CHAPTER 6. DIGITAL FILTER DESIGN PROGRAM SYSTEM

6-1 INTRODUCTION

Digital signal processing is concerned with the representation of signals by sequences of numbers or symbols and the processing of these sequences. Historically, digital signal processing has been done by the high speed digital computer whose progress has stimulated the developments of the complex and sophisticated signal processing algorithms. By the recent advances in integrated circuit technology, it is promised that economical implementations of very complex digital signal processing systems in which digital signal is processed in real time, become possible.

In digital signal processing systems, digital filter performs essential roles. Therefore, many studies on digital filter have been contributed in recent few years.

In this chapter, we describe the newly developed digital filter design program system called DINETS (DIGital NETwork design program System).^{(35) (36) (37)} First, the fundamental concept on digital filter design and some related important problems are explained. Secondly, the DINETS system is described along with the programs of three phases : approximation, synthesis and analysis. Finally, an actual digital filter design example applied by means of DINETS, to transmultiplexer is shown, which is a typical application system of digital signal processing.

6-2 GENERAL CHARACTERISTICS ON DIGITAL FILTER^{(38) (39) (40)}

6-2-1 Digital Signal Processing

Digital filter can be used for both digital and analog systems. When applied to analog continuous system, the block diagram and the principle are drawn in Fig. 6-1.

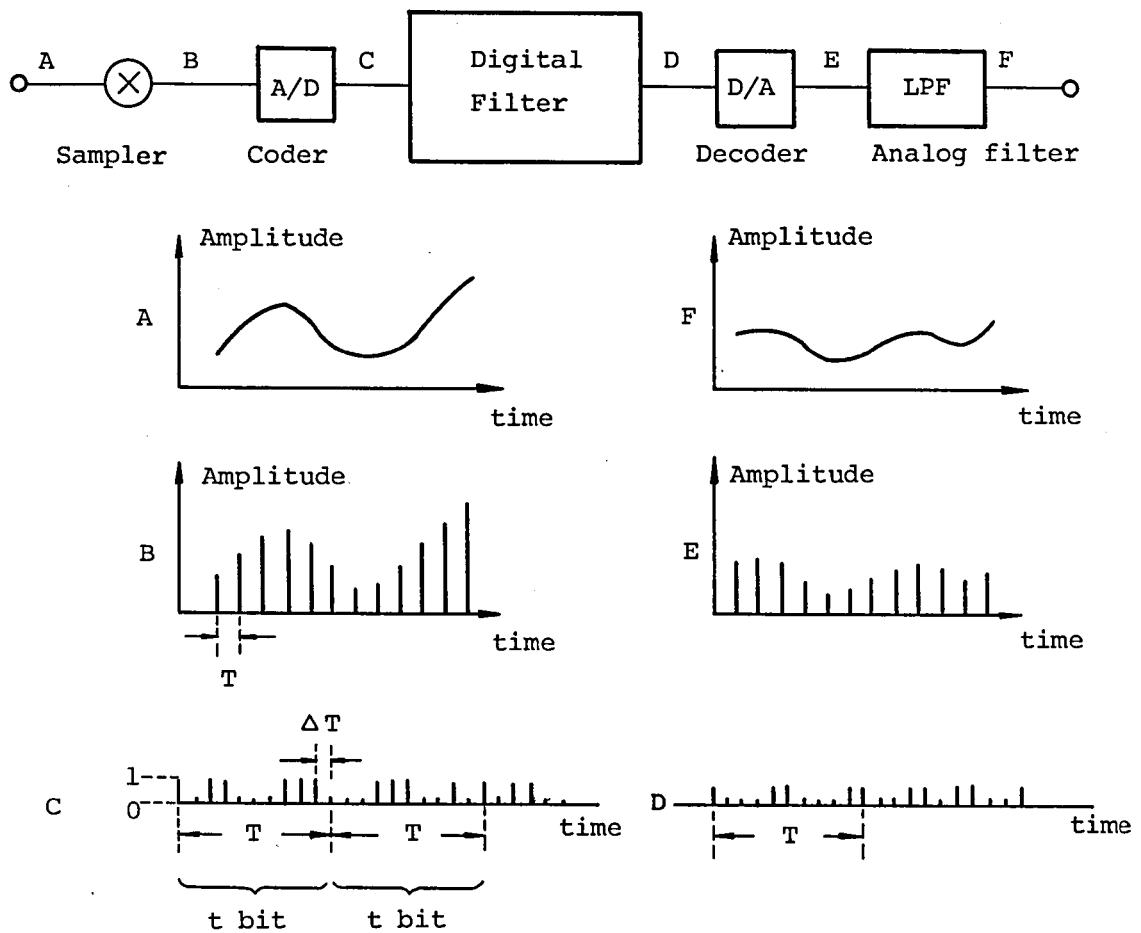


Fig. 6-1 System block diagram of digital filter applied to analog system.

In Fig. 6-1, an analog continuous band limited signal(A) is transformed into analog sampled signal(B) by a sampler, whose sampling rate T is determined by the sampling theorem. The signal(B) is converted to t -bit digital signal(C) by A/D converter within T second conversion time. Digital filter handles signal(C) and then the digitally processed signal(D) is put into D/A converter which converts digital signal(D) into analog sampled signal(E). Finally analog filter produces analog continuous signal(F) by smoothing analog sampled signal(E).

Here we can see that, in this system, quantization and recovery errors occur by A/D and D/A converters, in which analog signal is expressed by t -bit digital signal. If we handle with digital signal, we do not need to consider these errors. For this reason, in the following discussions, we are interested in digital signal only.

Digital signal expressed with finite word length is arithmetically processed in digital filter by means of three fundamental elements: adder, delay element, and multiplier which are denoted in Fig. 6-2.

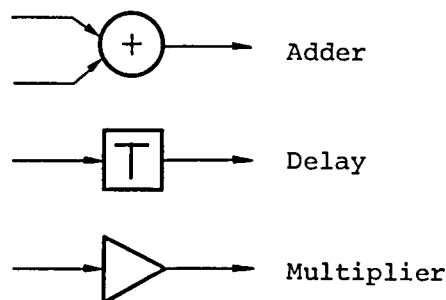


Fig. 6-2 Three fundamental elements of digital filter.

Each element is composed of logic devices. For example, a multiplier which is the most important element in digital signal processing, have a configuration as shown in Fig. 6-3.

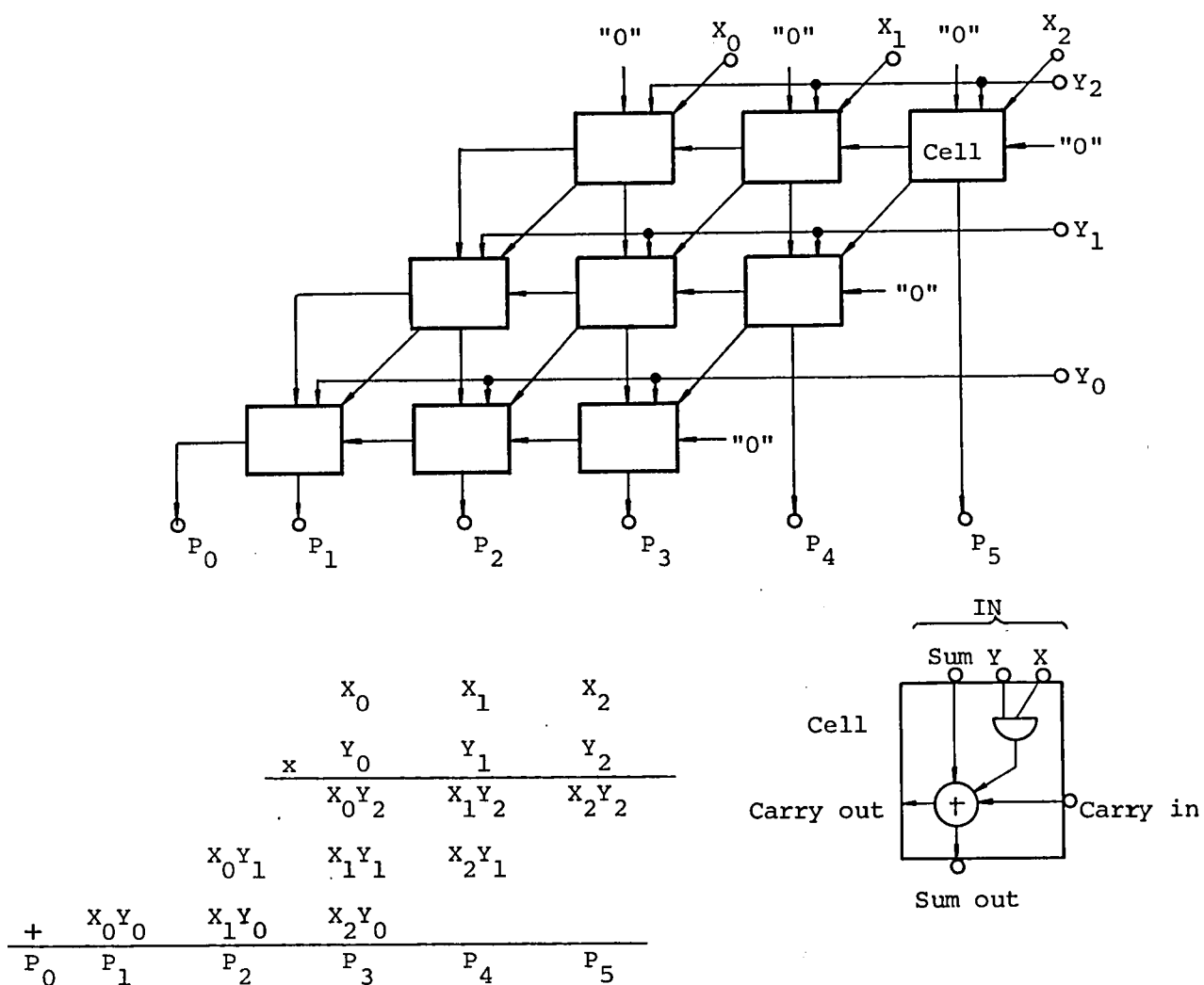


Fig. 6-3 An example of multiplier circuit.

This figure shows an example of parallel multiplier in which multiplicand is expressed in 3 bits and multiplier is also

expressed in 3 bits. Each cell in the figure consists of AND gate logic circuits.

6-2-2 Features of Digital Filter

Digital filter has several special features in the following points listed below.

- (1) No electrical adjustment process is needed in manufacturing stage, which produces the good uniformity of filter characteristics in mass production.
- (2) A filter can be realized with exactly the same characteristics as the designed one.
- (3) By using LSI technology, super-miniaturized filter can be obtained.
- (4) A filter with variable characteristics can be easily realized by means of read only memory (ROM).
- (5) Using an universal hardware implementation block, the time multiplexing and function multiplexing are possible.
- (6) Very high performances can be easily obtained, which are impossible with analog filters.

These features are extremely attractive not only for filter engineers but also for production engineers.

6-2-3 Sampled System Representation

Since digital filter is expressed in a sampled system, it is mathematically represented by a N-th order constant coefficient linear difference equation of the form

$$\sum_{k=0}^N a_k y(n-k) = \sum_{r=0}^M b_r x(n-r) \quad (6-1)$$

for the input $x(n)$ and the output $y(n)$.

Here, the z-transform $X(z)$ of a sequence $x(n)$ is defined by

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (6-2)$$

The z-transform of Eq.(6-1) becomes

$$\sum_{k=0}^N a_k z^{-k} Y(z) = \sum_{r=0}^M b_r z^{-r} X(z)$$

Therefore, we have a transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{r=0}^M b_r z^{-r}}{\sum_{k=0}^N a_k z^{-k}} \quad (6-3)$$

$H(z)$ expresses frequency characteristics in digital filter, which are evaluated along the unit circle $z=e^{j\omega T}$ in the z-plane, while frequency characteristics in analog filter are evaluated along the imaginary axis $p=j\Omega$ in the p-plane.

This tells us that digital filter has periodic frequency characteristics whose period is $2\pi/T$.

Next, we consider the stable condition together with an example. Let us consider an impulse response $h(n)=a^n U(n)$ where $U(n)$ expresses unit step sequence. The corresponding z-transform is

$$H(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \frac{1}{1-az^{-1}}, \text{ if } |z| > |a|. \quad (6-4)$$

The condition $|z| > |a|$ is called region of convergence. Since we evaluate $H(e^{j\omega T})$ along the unit circle, region of convergence must include the unit circle, which is equivalent to $|a| < 1$. This suggests us, as the stable condition, that the poles in $H(z)$ must exist inside the unit circle. These are shown in Fig. 6-4, in which the shaded region expresses the region of convergence. In contrast, the poles in $H(p)$ in analog filter must exist in the left half plane in the p-plane.

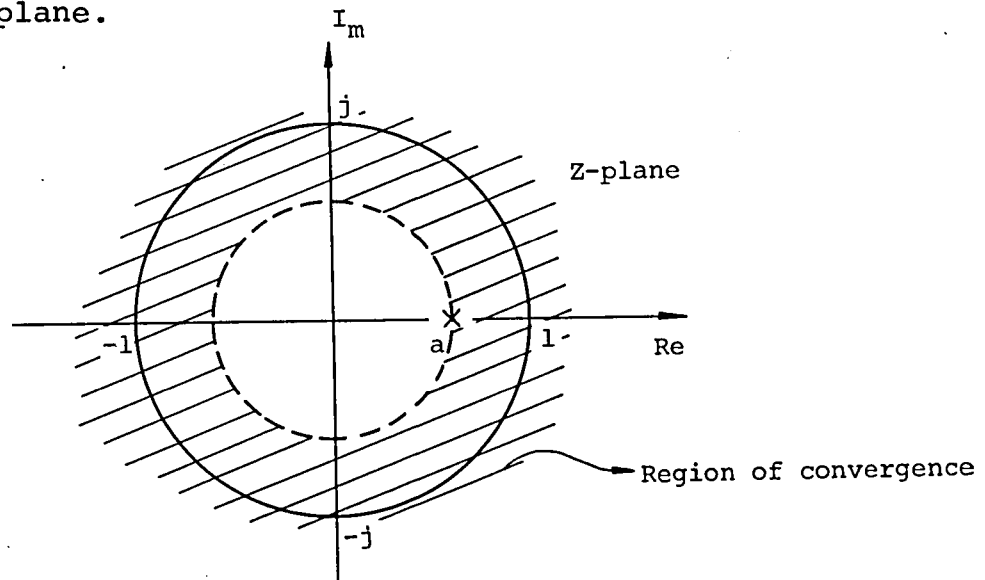


Fig. 6-4 The stable condition of digital filter for Eq.(6-4).

6-2-4 IIR and FIR Digital Filters

If the denominator in Eq.(6-3) is unity, $H(z)$ will be referred to as a finite impulse response (FIR) filter because the unit-sample response of FIR filter is of finite duration. If the denominator in Eq.(6-3) is not unity, $H(z)$ will be referred to as an infinite impulse response (IIR) filter because the unit-sample response of IIR filter is of infinite duration.

An FIR filter has many advantages such that it can have precisely linear phase and it does not need to consider the stable condition etc.

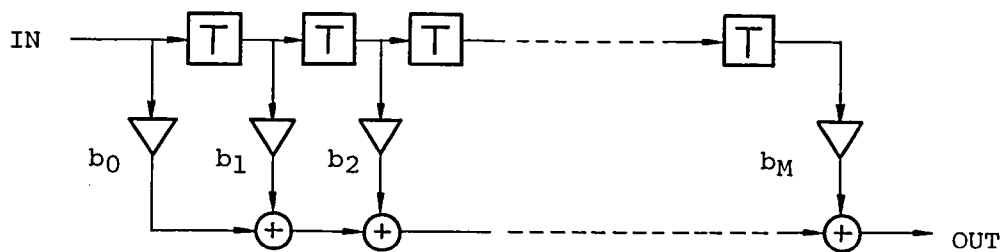
In contrast, if we neglect phase consideration, it is generally true that a prescribed amplitude specification will be met most efficiently with an IIR filter.

Thus the choice of an FIR filter or an IIR filter depends on the circumstances.

Next, we consider an FIR filter and an IIR filter implementations.

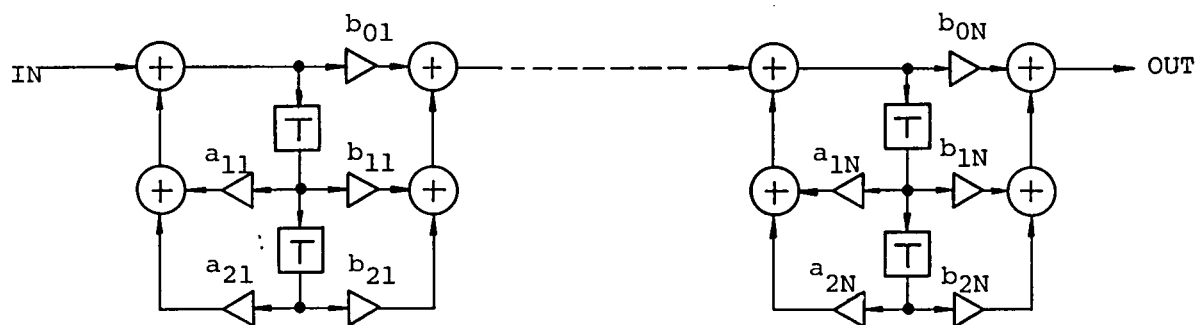
An FIR filter is usually implemented by transversal filter form as shown in Fig. 6-5.

An IIR filter has usually three types of implementation: direct, parallel and cascade forms. However, it is generally true that the cascade form shown in Fig. 6-6 is most suitable from the standpoints of sensitivity and applicability to LSI.



$$H(Z) = \sum_{r=0}^M b_r Z^{-r}$$

Fig. 6-5 FIR filter implementation.



$$H(Z) = \prod_{k=1}^N \frac{b_{0k} + b_{1k} Z^{-1} + b_{2k} Z^{-2}}{1 + a_{1k} Z^{-1} + a_{2k} Z^{-2}}$$

Fig. 6-6 IIR filter cascade form implementation.

6-2-5 Effect of Finite Word Length

From the view points of hardware complexity, fixed point representation is usually adopted as the type of arithmetic. So, the fixed point arithmetic is used throughout this chapter.

As a result of the finite word length in digital filter, each coefficient is replaced by its t-bit representation. That is, the coefficient a_k is replaced by $a_k + d_k$ with d_k bounded in absolute value by 2^{-t} . Therefore, the filter characteristics are changed.⁽⁴¹⁾ This tells us that, even though the original filter designed with infinite word length satisfies a given specification, the actual filter with t-bit rounded coefficients may not satisfy a given specification. One of the solutions of this problem is given by the newly developed design method which will be described in Section 6-3-3.

Next, we consider the round off noise. When s-bit state variable (A) in digital filter is multiplied by t-bit coefficient (B), the output variable of multiplier is rounded by s-bit (C) as shown in Fig.6-7. This rounding error is observed as noise. If the state variable is restricted lower than unity by scaling, then, multiplication does not cause overflow but addition causes overflow.

The spectrum of the output round off noise can be calculated by the form^{(42) (43)}

$$N(e^{j\omega T}) = \sigma^2 \sum_j k_j' |G_j'(e^{j\omega T})|^2 \quad (6-5)$$

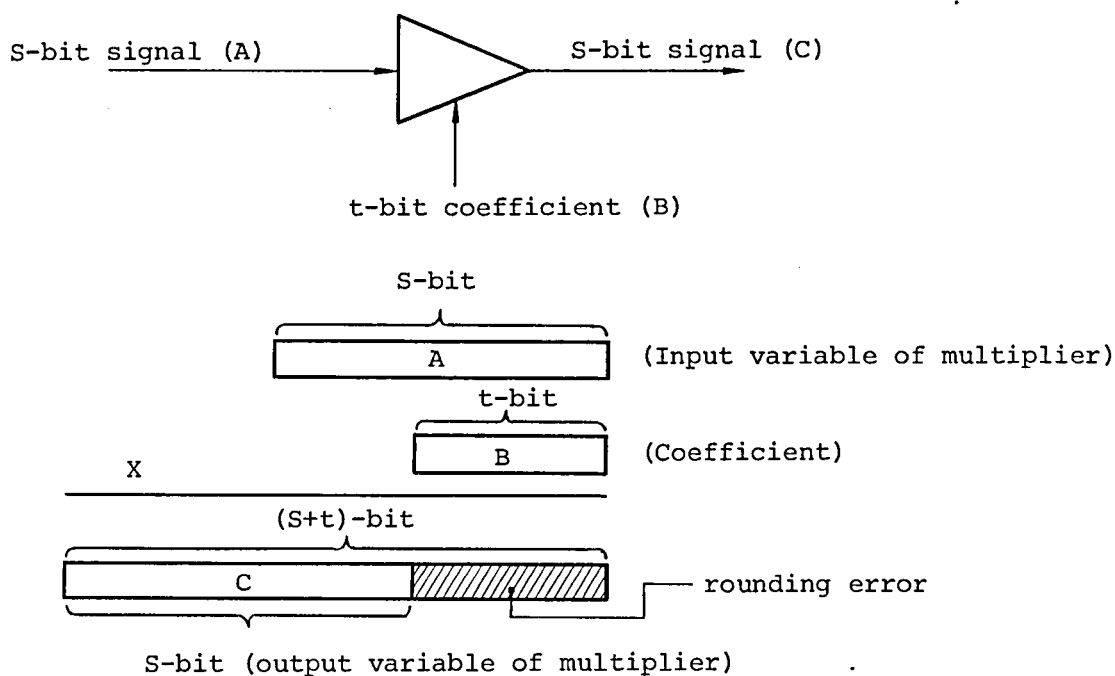


Fig. 6-7 Rounding error by multiplier.

where the $G_j^!(e^{j\omega T})$ are scaled transfer function from certain summation nodes in the digital circuit to the filter output. σ^2 is the variance of the rounding errors from each multiplier, and $k_j^!$ is an integer which indicates the number of error inputs to the respective summation nodes. From Eq. (6-5), we can see that the round off noise is affected by $G_j^!(e^{j\omega T})$ which is determined by the filter structure.

The effect of overflow after addition can be loosened by putting an overflow-correction at the output of adder. Fig. 6-8 shows an operation of saturation type overflow correction.

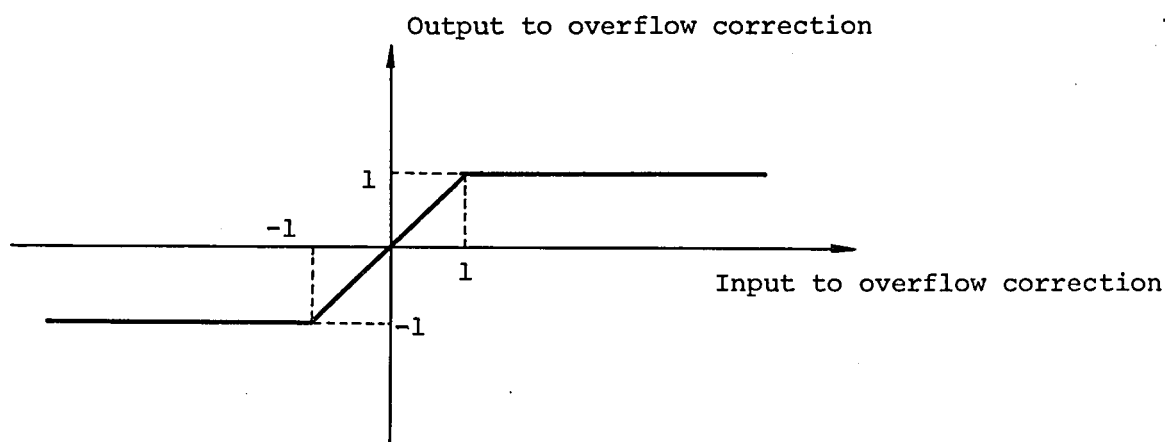


Fig. 6-8 Saturation type overflow correction.

An actual example of overflow and overflow correction will be shown in Section 6-3-5.

Rounding or truncation of the result of arithmetic operation often causes another important quantization error referred to as limit cycle oscillation. We can see it in a simple example. Let us consider a first order IIR filter of a form

$$y(n) = -\frac{1}{2}y(n-1) + x(n)$$

and

$$x(n) = 0, y(0) = 3/4.$$

If we can have the infinite word length at the output of multiplication, $y(n)$ will decay toward zero in the form

$$y(n) = \frac{3}{4} \left(-\frac{1}{2}\right)^n U(n).$$

But, if we may have three bits (i.e., a sign bit to the left of the binary point and two bit to the right of the binary point), the product $-\frac{1}{2} \cdot y(n-1)$ must be rounded or truncated in three bits. Assuming rounding of the product, this process will be shown in $y(n)$ with time sequences as follows:

$$\begin{aligned}
 n = 0 \quad y(0) &= \frac{3}{4} \\
 n = 1 \quad y(1) &= -\frac{1}{2}\left(\frac{3}{4}\right) = -\frac{3}{8} \xrightarrow{\text{rounding}} -\frac{1}{2} \\
 n = 2 \quad y(2) &= -\frac{1}{2}\left(-\frac{1}{2}\right) = \frac{1}{4} \xrightarrow{\text{rounding}} \frac{1}{4} \\
 n = 3 \quad y(3) &= -\frac{1}{2}\left(\frac{1}{4}\right) = -\frac{1}{8} \xrightarrow{\text{rounding}} -\frac{1}{4} \\
 n = 4 \quad y(4) &= -\frac{1}{2}\left(-\frac{1}{4}\right) = \frac{1}{8} \xrightarrow{\text{rounding}} \frac{1}{4} \\
 &\vdots
 \end{aligned}$$

Thus, $y(n)$ reaches a periodic oscillation $\frac{1}{4}$ or $-\frac{1}{4}$. This can be also observed as noise. In order to make the amplitude of a limit cycle small, we must have a longer word length.

6-3 DIGITAL FILTER DESIGN PROGRAM SYSTEM

6-3-1 Design Flow Chart

The flow chart of DINETS developed in this study is shown in Fig. 6-9. This system is organized with three phases of approximation, synthesis and analysis. In the approximation phase, the transfer function is designed with infinite coefficients word length, and these coefficients are optimized through discrete optimization process. In the synthesis phase, an

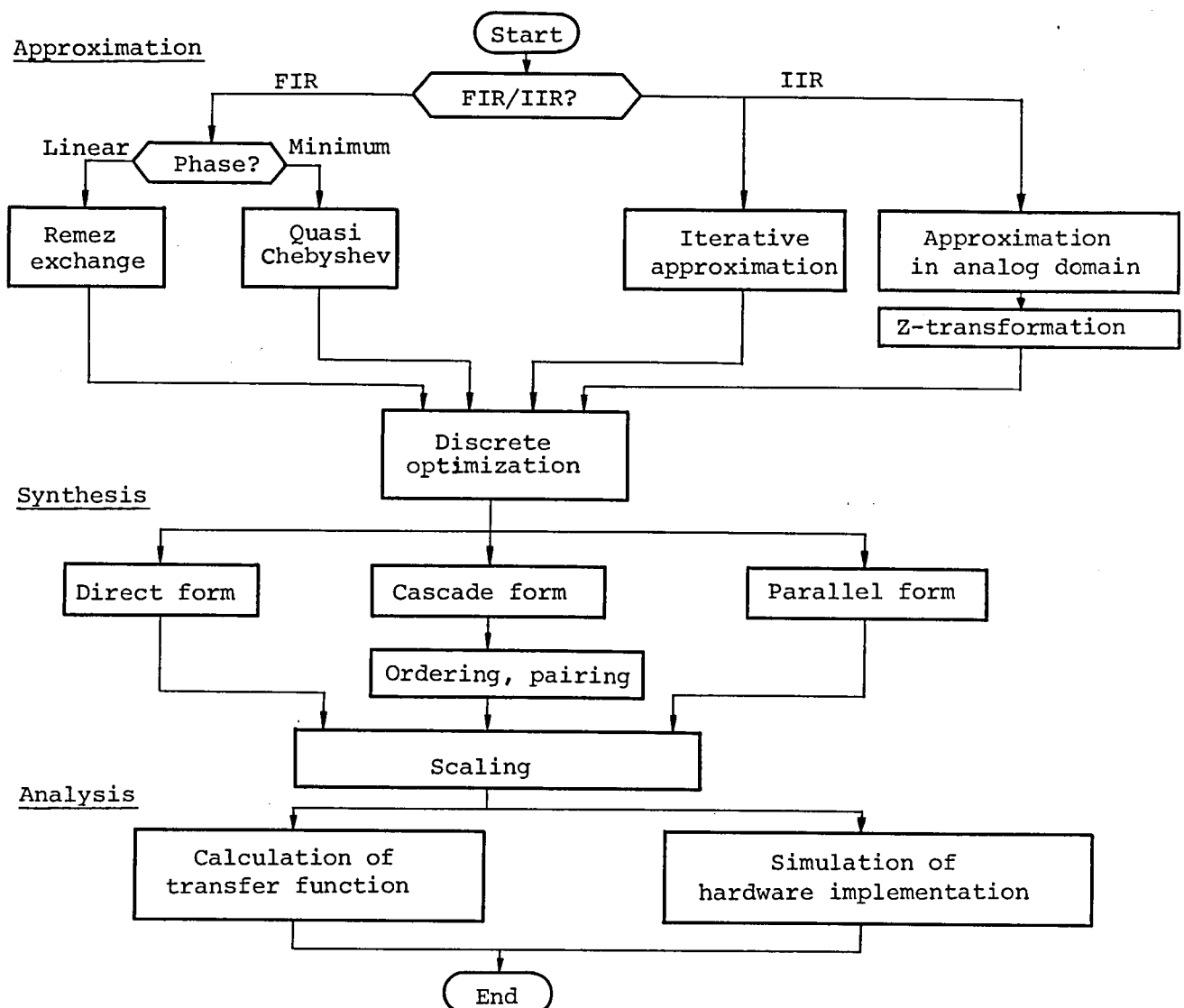


Fig. 6-9 The flow chart of DINETS.

optimum filter structure is selected to minimize the round off noise. In the analysis phase, the response of transfer function and the actual hardware implementation are simulated.

6-3-2 Approximation with Infinite Word Length

The weighted Chebyshev approximation of IIR filter with an arbitrary stop band and pass band characteristics, is carried out in analog frequency domain through pre-warping. For this step, the approximation theory stated in Chapter 2 together with its program can be fully applied. The transfer function can be converted into the z-plane by one of the three commonly used method: standard, matched or bilinear transformation.

Linear phase FIR filter is designed by the Remez exchange method.^{(44) (45)} Minimum phase FIR filter is designed by using the transfer function having quasi-equiripple characteristics in the stop band.⁽⁴⁶⁾

An iterative Chebyshev approximation program⁽³²⁾ is also prepared for the filter shaping in the pass band and, in addition, this program can be applied to a time domain design of IIR filter which will be described in detail in Chapter 7.

6-3-3 Approximation with Finite Word Length^{(47) (48) (49) (50) (51) (52) (53)}

If a given specification is not met because of the deviation of the filter response caused by rounding coefficients, it is necessary to optimize the coefficients in finite precision. The optimization procedure is fundamentally based on the random search method.^{(48) (53)}

Let X be a N dimensional initial coefficient set
 $X = (x_1, x_2, \dots, x_N),$

then a new coefficient set X' are generated from X such that

$$X' = X + \Delta X$$

where $\Delta X = (\Delta x_1, \Delta x_2, \dots, \Delta x_N)$.

X' is generated randomly according to a specified probability distribution function. Scanning area of ΔX is given previously, and scanning interval is chosen to be equal to quantization step size of coefficients.

Our approach is carried out through the following two steps. In step 1, the rounded values of original coefficients which are obtained with infinite precision, are chosen as the initial set. Generation of random number is specified by a uniform distribution function. Moving the coefficients set uniformly in the scanning area, the transfer function is evaluated. The coefficient set which shows the best characteristics is chosen as the initial set for step 2 as shown in Fig. 6-10. In step 2, normal distribution function is used to generate the random number, and the scanning of optimum solution is concentrated in the vicinity of the initial set. Through these two steps, the final optimum coefficient set is obtained.

In view of computing time or the number of transfer function evaluation, the optimization procedure through the above two steps is superior to the conventional one which used the first step or the second step only. 50% reduction of the number of evaluation can be achieved. One example is

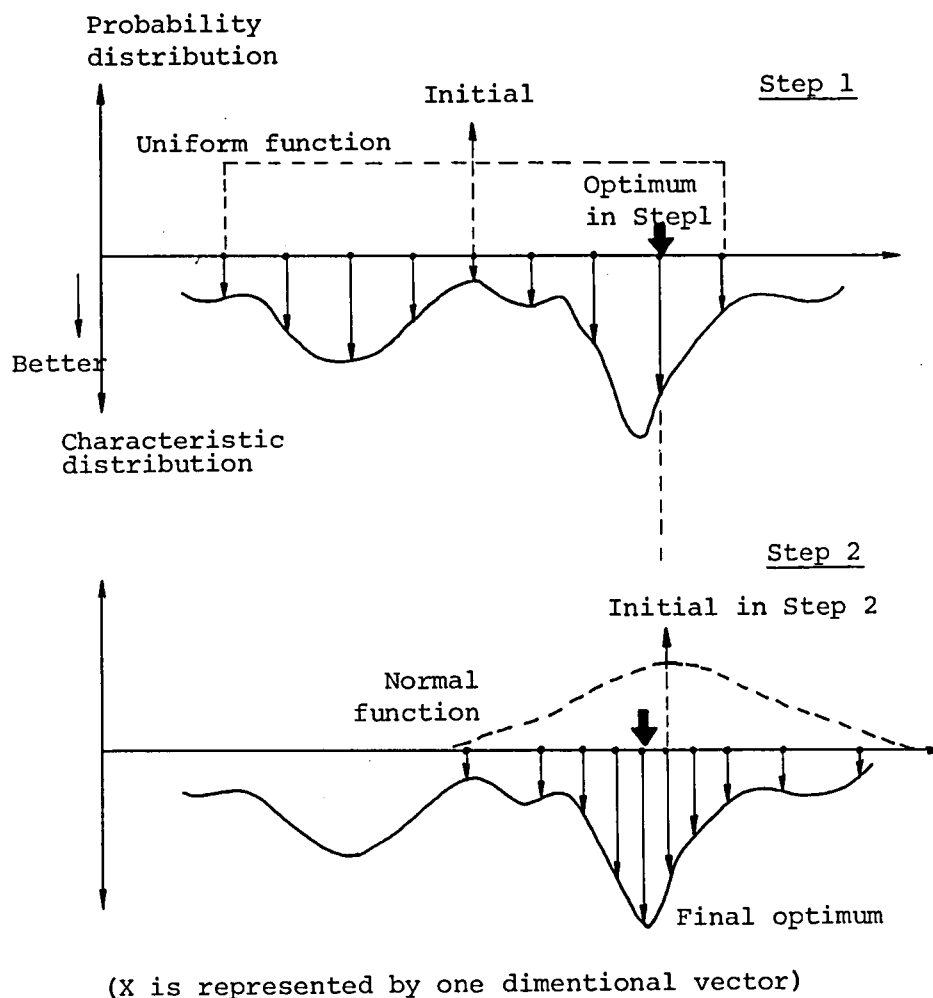


Fig. 6-10 Optimization procedure through two steps.

shown in Fig. 6-11. This figure shows the loss characteristics of 8th degree LPF with 12-bit word length. Through the proposed optimization procedure, the word length of coefficients can be reduced by 3 bits compared with rounding only. In other words, in the case of rounding only, 15-bit word length is required in order to satisfy the specification.

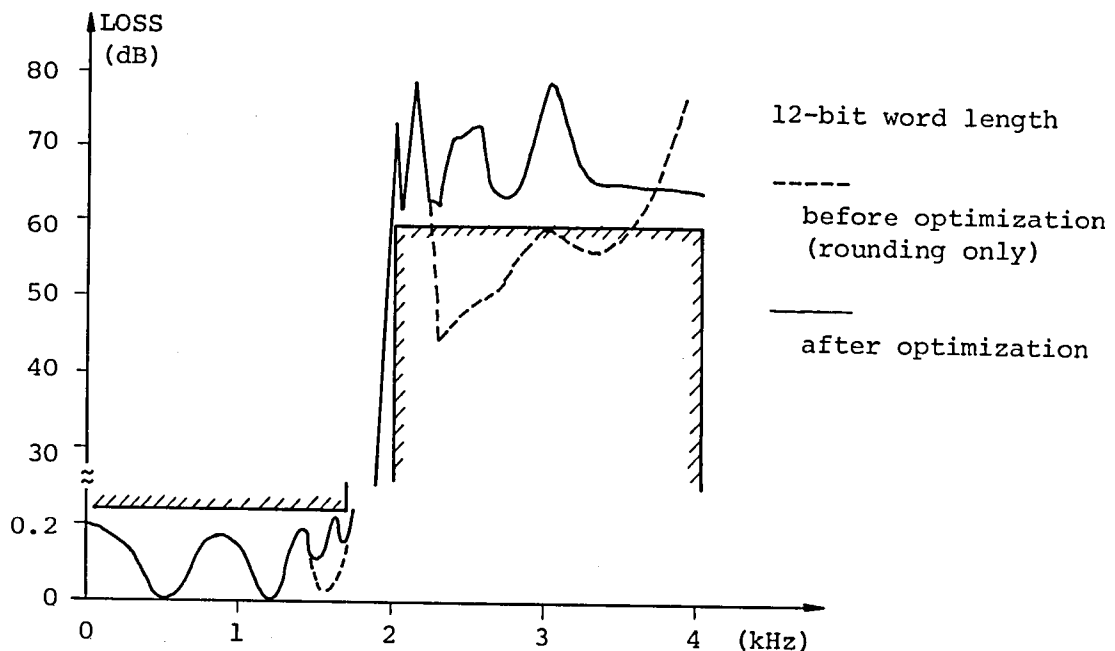


Fig. 6-11 Comparison of loss characteristics between before and after optimization.

6-3-4 Synthesis^{(54) (55) (56) (57) (58) (59) (60)}

In the synthesis phase, the filter structure having low round off noise with considering its dynamic range constraint, is selected through the optimization procedure. In the program, the scaling factor which is closely related to the dynamic range constraints, is determined and the round off noise is calculated for three commonly used digital filter structures, direct, parallel and cascade forms.

In the cascade form realization, the round off noise is highly dependent on the ordering and pairing of poles and zeros. Therefore, for some set of ordering and pairing, the

total round off noise is calculated by integrating Eq.(6-5) over the finite frequency range and, then, a digital circuit structure which shows minimum total round off noise, is chosen. A calculation example of noise spectrum by Eq. (6-5) for 8th degree cascade structure LPF whose passband is from DC to 1.7 kHz, is shown in Fig. 6-12.

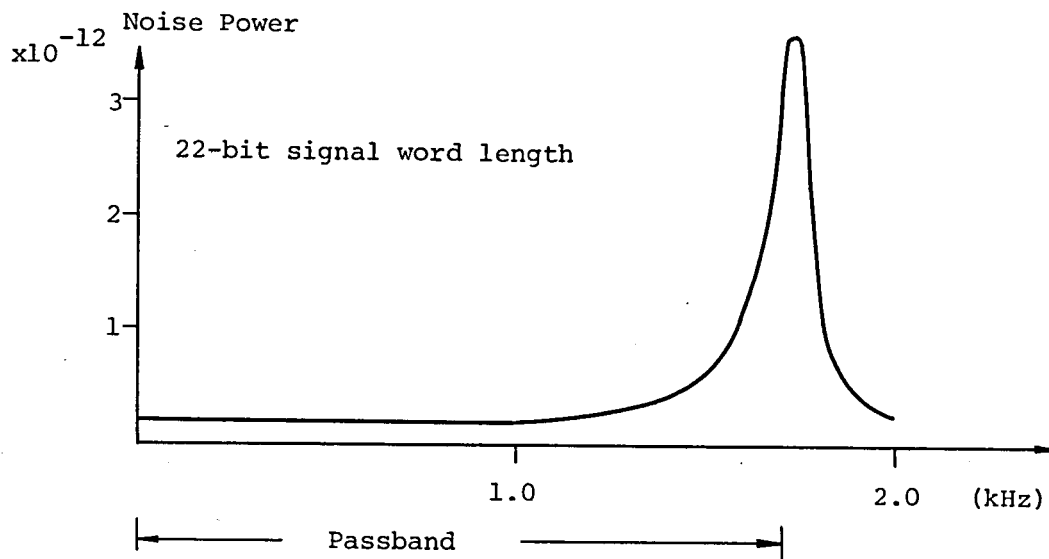
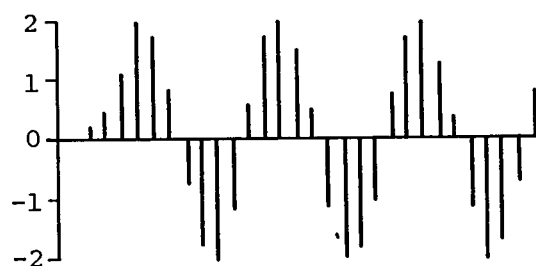


Fig. 6-12 An example of noise spectrum.

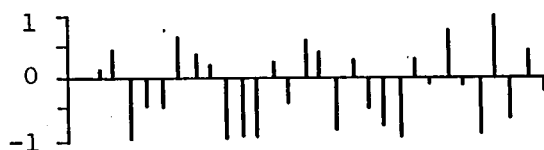
6-3-5 Analysis^{(61) (62)}

There are two kinds of analysis programs in DINETS. In the first analysis program, the frequency response (amplitude, loss, phase and delay) can be calculated whose example will be shown in Fig. 6-15 which will appear in Section 6-4.

In the second program, an actual hardware implementation of digital filter can be simulated. This simulation program performs completely the same operations as the actual hardware operations such as rounding of data and overflow correction along with time sequences in the network. The examples of the time response analyzed by the program are shown in Fig. 6-13. Fig. 6-13-a shows the time response with infinite precision, Fig. 6-13-b shows the one with finite word length in which overflows occur, and Fig. 6-13-c shows the one with overflow correction, in which errors are reduced.



(a) Infinite word length



(b) Finite word length
without overflow correction



(c) Finite word length
with overflow correction

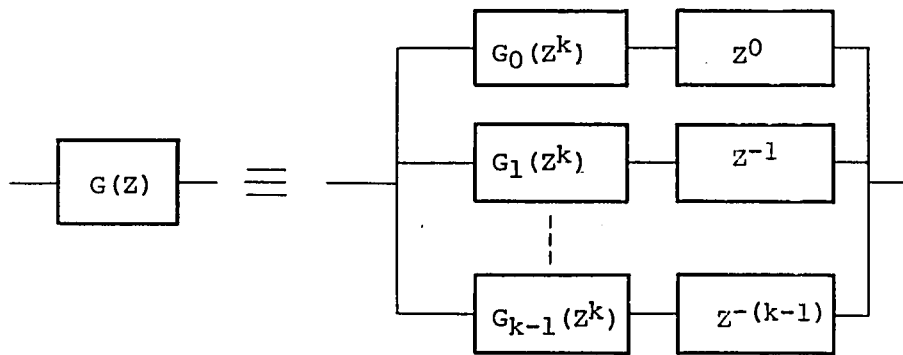
Fig. 6-13 Output signal for sine wave input signal.

6-4 DESIGN EXAMPLE

Digital filter applied to transmultiplexer which is used to interconnect analog and digital communication networks at the multiplex level⁽⁶³⁾⁽⁶⁴⁾⁽⁶⁵⁾, is shown as an example. In this system, digital filter transfer function has a form

$$G(z) = \sum_{i=0}^{k-1} z^{-i} \cdot G_i(z^k), \quad z = e^{j\omega T} \quad (6-6)$$

whose construction is illustrated in Fig. 6-14.



Equivalent realization of $G(Z)$ by $G_i(Z^k)$

Fig. 6-14 Illustration of Eq.(6-6).

$G(z)$ operates at a rate of $(1/T)$ Hz while $G_i(z^k)$ operates at a rate of $(1/kT)$ Hz.

The actual design parameters are listed below.

- Coefficient word length = 14 bit
- Signal (state variable) word length = 36 bit

- Sampling rate of $G(z) = \frac{1}{T} = 224 \text{ kHz}$
- Sampling rate of $G_1(z^k) = \frac{1}{kT} = 8 \text{ kHz}$
- $k = 28$ (28 time division multiplex)
- Clock rate $= \frac{1}{\Delta T} = 8 \text{ kHz} \times 28 \times 36 = 8.064 \text{ MHz}$

$G_1(z^k)$ is realized by 8th degree cascade form IIR filter. The frequency characteristics of $G(z)$ is shown in Fig. 6-15. A photograph of an actual hardware implementation is also shown in Fig. 6-16.

6-5 CONCLUSION

We discussed briefly the general concept on digital filter design and then, described the newly developed digital filter design program system DINETS. Throughout the course of the design of DINETS program, the strong attention was paid to the saving of computation time.

DINETS was successfully developed very timely because any kind of systems, not only for communication systems, will be digitalized in very near future. However many unsolved problems, in particular, related to quantization errors caused by finite word length, are still remained. DINETS, therefore, will be a step for a more powerful design program system.

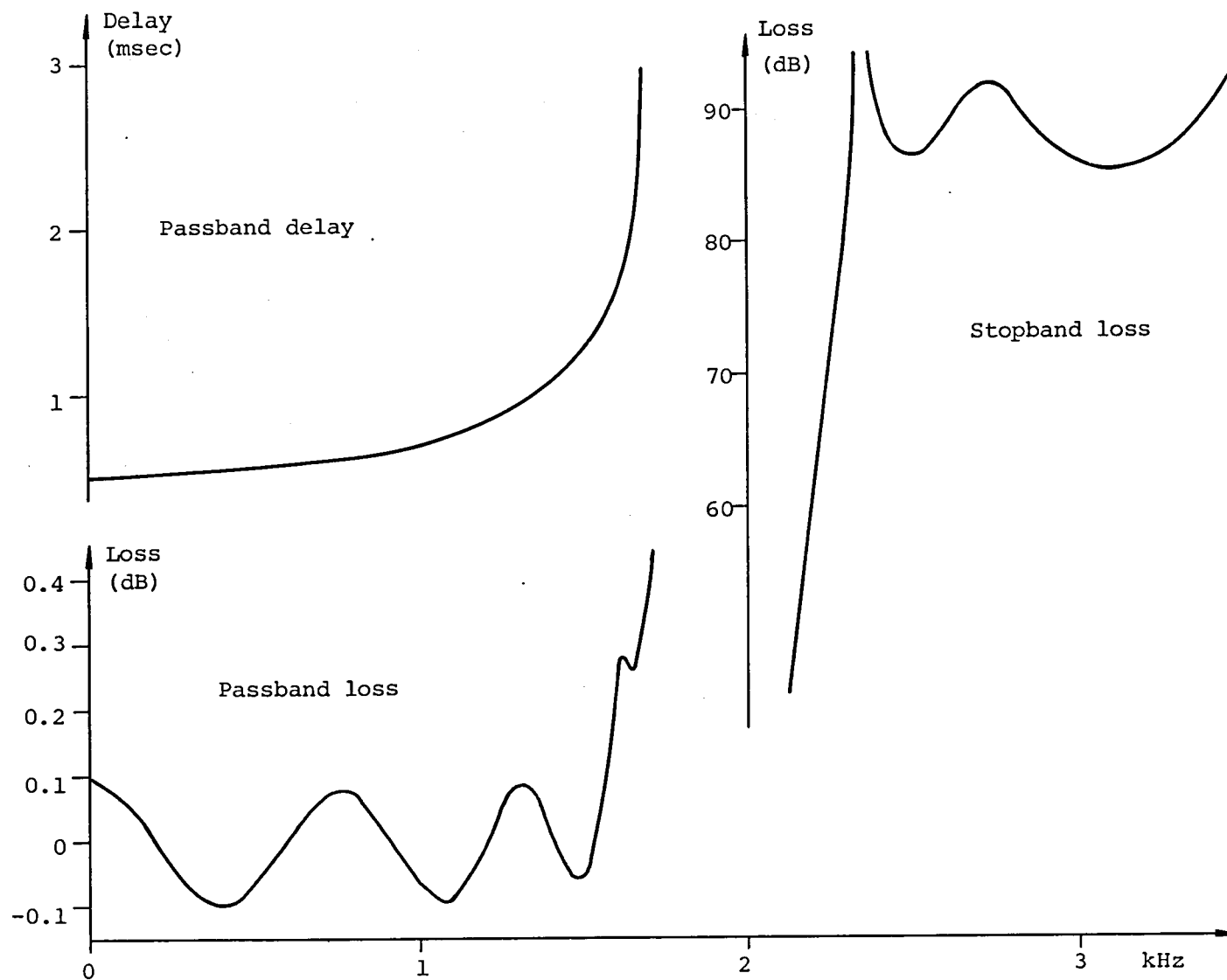
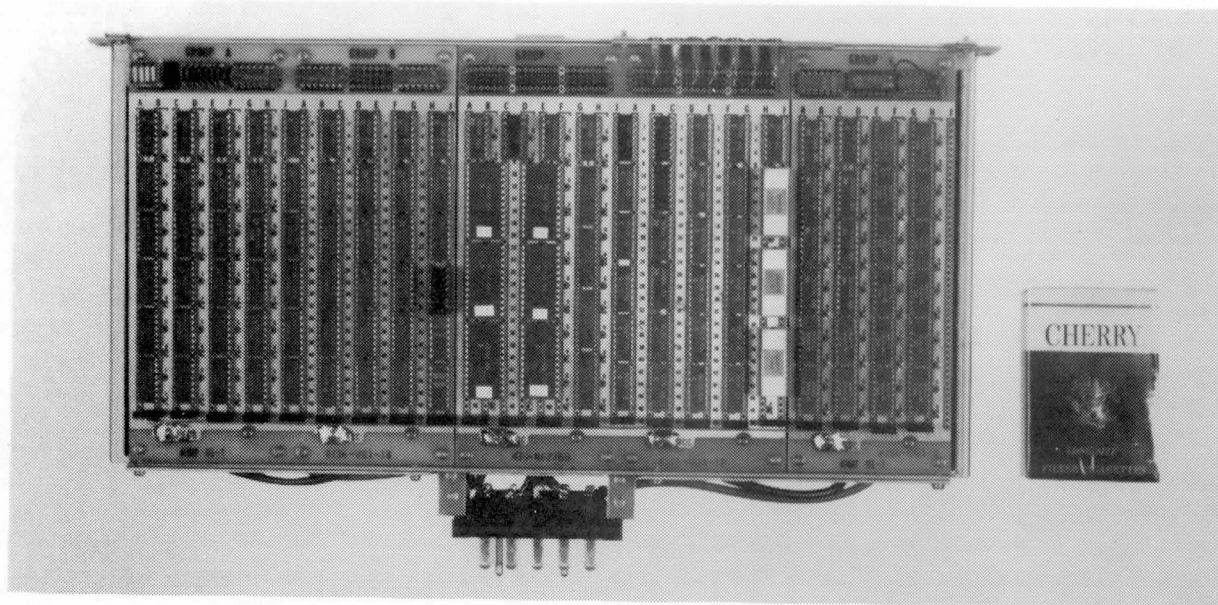
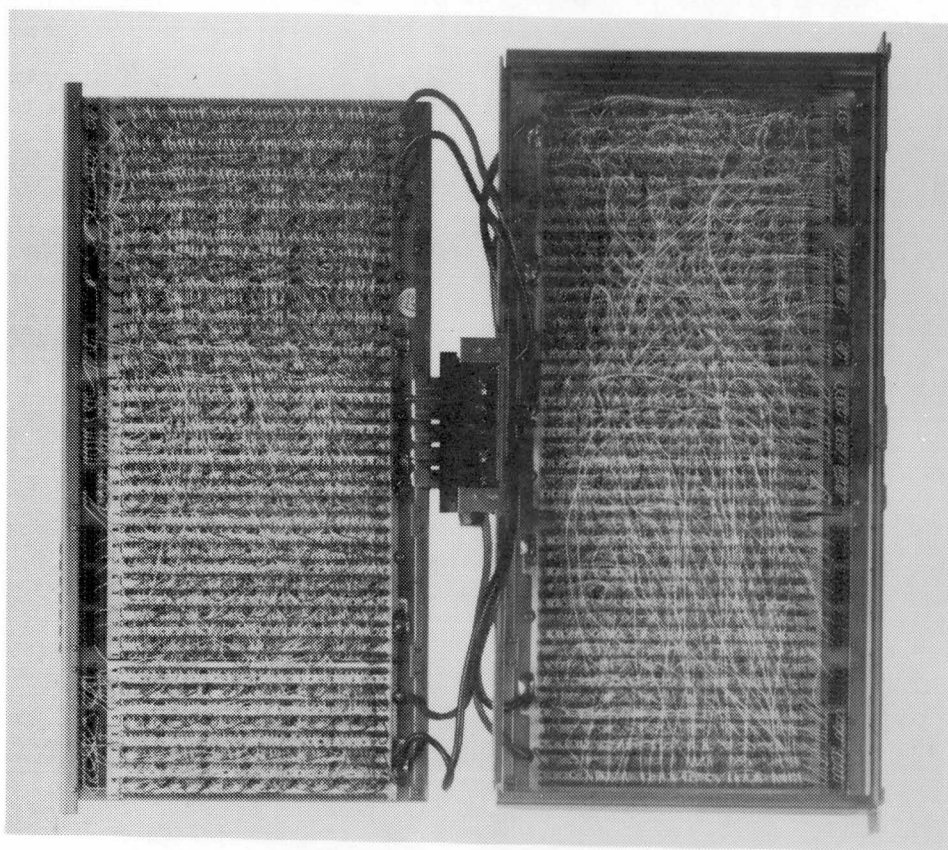


Fig. 6-15 Frequency characteristics of $G(z)$.



(a) Circuit board for LSI's (front)



(b) Circuit board for wire wrapping (back)

Fig. 6-16 Digital filter hardware implementation.

CHAPTER 7. IIR LOW PASS FILTERS WITH SPECIFIED EQUIRIPPLE STOPBAND LOSS AND ITS TIME RESPONSE APPROXIMATION

7-1 INTRODUCTION

One of the important problems in designing pulse transmission systems such as pulse code modulation (PCM) transmission system and data transmission system, is to design the pulse shaping network which satisfies the specifications in both the frequency domain and time domain simultaneously, where the frequency domain specification is related to the interference for adjacent channels and the time domain specification is related to the intersymbol interference. ^{(66) (67) (68) (69) (70)} This network is actually realized by a low pass filter such that it satisfies the given specified stopband attenuation loss in the frequency domain while the intersymbol interference becomes zero in the time domain.

In a continuous system, a low pass transfer function having Chebyshev characteristic in the stopband was given by K.H. Feistel and R.Unbehauen.⁽⁷¹⁾ By using this transfer function, G.C.Temes, M.Gyi⁽⁷²⁾ and D.A. Spaulding⁽⁷³⁾ gave the time response approximation method through iterative optimization procedure.

N.Yoshida and Y.Ishizaki^{(74) (75)} presented a stopband Chebyshev transfer function which preserves a prescribed stopband minimum effective attenuation loss in a constant while the time response is iteratively optimized. This improves the

method of G.C.Temes and M.Gyi in which, in the course of approximation process, the attenuation loss at zero frequency varies and as a result, the given stopband effective attenuation specification is not satisfied.

In a sampled system, for FIR and IIR filters, S.Tsujii et al^{(76) (77) (78) (79)} give a method of time response approximation, preserving the effective stopband attenuation in a constant within a given tolerance by using a linear programming problem technique.

In this chapter, a stopband equiripple transfer function in a sampled system is obtained by using the method of K.H. Feistel and R.Unbehauen. In addition, the condition such that a stopband equiripple transfer function has a prescribed minimum stopband loss, is considered here, applying the method of N.Yoshida and Y.Ishizaki and, then, we introduce the design theory for time response approximation which satisfies this condition.^{(80) (81)} Finally, the design examples applied to Nyquist pulse shaping network are shown.

7-2 STOPBAND EQUIRIPPLE TRANSFER FUNCTION IN A SAMPLED SYSTEM

In a continuous system which is expressed in the p -plane, a stopband equiripple low pass transfer function is given by K.H. Feistel, R.Unbehauen⁽⁷¹⁾ and N.Yoshida, Y.Ishizaki⁽⁷⁴⁾ as follows:

$$H(p) = K \frac{G_e \left[\prod_{i=1}^n (\zeta + \zeta_i) \right]}{\prod_{i=1}^n (p - p_i)} \quad (7-1)$$

where

$$\zeta = \sqrt{p^2 + \Omega_c^2} \quad \text{Re} [\zeta] > 0 \quad (7-2)$$

$$\zeta_i = \sqrt{p_i^2 + \Omega_c^2} \quad \text{Re} [\zeta_i] > 0$$

and K expresses a positive constant. G_e means to take the even part of polynomial.

In $p=j\Omega$, $|H(j\Omega)|$ have a equiripple amplitude characteristics between 0 and K in the frequency range $|\Omega_c| \leq \Omega$ in which Ω_c means the stopband edge frequency. The transformation from p-plane to z-plane in which a sampled system is expressed, is performed by a following relation.

$$p = d \frac{1 - z^{-1}}{1 + z^{-1}} \quad \text{and} \quad z = \frac{d + p}{d - p} \quad (7-3)$$

where $z = e^{j\omega T}$, $T = 1/f_s$, f_s and d express the sampling frequency and the positive constant, respectively. When ω_c is taken to be the stopband edge frequency in the z-plane, from Eq.(7-3), we have

$$\begin{aligned}
e^{j\omega_c T} &= \frac{d + j\Omega_c}{d - j\Omega_c} \\
&= \frac{1 - t^2}{1 + t^2} + j \frac{2t}{1 + t^2}
\end{aligned} \tag{7-4}$$

where $t = \Omega_c/d$. From Eq.(7-4),

$$\cos\omega_c T = \frac{1 - t^2}{1 + t^2} \quad \text{and} \quad \sin\omega_c T = \frac{2t}{1 + t^2} . \tag{7-5}$$

The transformation by Eq.(7-3) means that the imaginary axis in the p-plane shown in Fig. 7-1-a is mapped to the unit circle in the z-plane in Fig. 7-1-b.

In these figures, $\sim\sim\sim$ indicates the frequency range of equiripple amplitude. Eq.(7-2) is squared and is substituted by the relation (7-3). We have

$$\zeta^2 = d^2(1 + t^2) \frac{1 - 2 \frac{1 - t^2}{1 + t^2} z^{-1} + z^{-2}}{1 + 2z^{-1} + z^{-2}} . \tag{7-6}$$

The denominator in Eq.(7-1) can be rewritten by the substitution of Eq.(7-3).

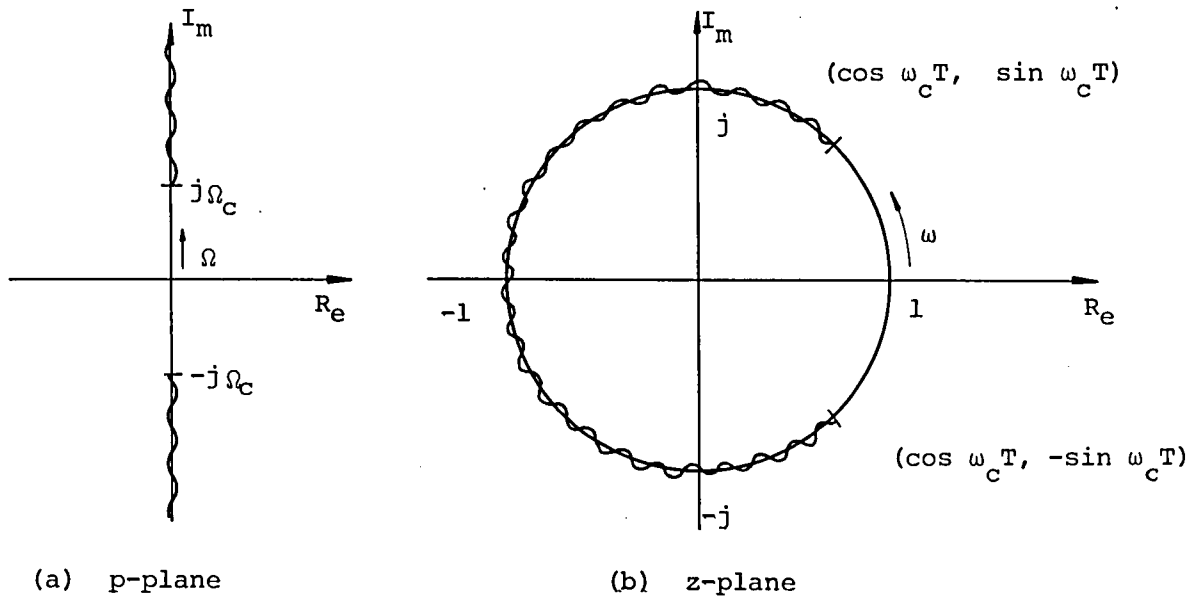


Fig. 7-1 The transformation from the p-plane to the z-plane.

$$\prod_{i=1}^n (p-p_i) = \prod_{i=1}^n \left[2d \frac{1-z_i z^{-1}}{(1+z_i^{-1})(1+z_i)} \right] \quad (7-7)$$

From Eq.(7-5),

$$1 + t^2 = \frac{2}{1 + \cos \omega_c T} = \frac{1}{\cos^2 \frac{\omega_c T}{2}} \quad .$$

Since $0 \leq \omega_c T \leq \pi$,

$$\sqrt{1+t^2} = \frac{1}{\cos \frac{\omega_c T}{2}} \quad (7-8)$$

By substituting Eq.(7-6) and Eq.(7-7) into Eq.(7-1) and using Eq.(7-8), we have an IIR low pass transfer function having equiripple stop band loss.

$$H(z^{-1}) = K \frac{1}{(2\cos \frac{\omega_c T}{2})^n} \cdot \frac{G_e \left[\prod_{i=1}^n (\zeta + \zeta_i) \right]}{\prod_{i=1}^n \frac{1 - z_i z^{-1}}{(1+z_i^{-1})(1+z_i)}} \quad (7-9)$$

where

$$\zeta^2 = \frac{1 - 2\cos \omega_c T \cdot z^{-1} + z^{-2}}{1 + 2z^{-1} + z^{-2}} \quad (7-10)$$

$$\zeta_i^2 = \frac{1 - 2\cos \omega_c T \cdot z_i^{-1} + z_i^{-2}}{1 + 2z_i^{-1} + z_i^{-2}} .$$

It is well known that, if we use the relation (7-2), all the zeros of numerator in Eq.(7-1) exist on the $j\Omega$ axis in the p -plane and exist in the frequency region of $|\Omega| > \Omega_c^{(5)(75)}$.

In this case, the numerator in Eq.(7-1) can be written as follows:

$$G_e \left[\prod_{i=1}^n (\zeta + \zeta_i) \right] = \prod_{i=1}^{\ell} (\zeta^2 + Q_i^2) \quad (7-11)$$

where Q_i^2 is positive, and $\ell = (n-1)/2$ for odd n , $\ell = n/2$ for even n . Here, in Eq.(7-9), we have a following theorem.

[Theorem 7-1]

In Eq.(7-10), if we have ζ_i as

$$\zeta_i = \sqrt{\frac{1 - 2\cos \omega_c T \cdot z_i^{-1} + z_i^{-2}}{1 + 2z_i^{-1} + z_i^{-2}}} \quad \text{and} \quad \text{Re}[\zeta_i] > 0, \quad (7-12)$$

all the zeros of numerator in Eq.(7-9) exist on the unit circle in the z-plane and exist in the frequency region of $|\omega| > \omega_c$.

(Proof) In the case that the theorem's condition is satisfied, the numerator in Eq.(7-9) can be written by the same form as Eq.(7-11). We substitute Eq.(7-10) into Eq.(7-11).

$$\text{Ge} \left[\prod_{i=1}^n (\zeta + \zeta_i) \right] = \prod_{i=1}^l \left(\frac{(1+Q_i^2)z^{-2} + 2(Q_i^2 - \cos\omega_c T)z^{-1} + 1 + Q_i^2}{1 + 2z^{-1} + z^{-2}} \right).$$

In the second order equation for z in the i-th term in numerator of right hand side, its discriminative equation D_i is expressed as follows:

$$\begin{aligned} D_i &= (Q_i^2 - \cos\omega_c T)^2 - (1 + Q_i^2)^2 \\ &= (\cos\omega_c T + 1)(\cos\omega_c T - 1 - 2Q_i^2). \end{aligned}$$

Since $Q_i^2 > 0$, we have $D_i < 0$.

Hence, z_{0i} , zeros of i-th term can be written as

$$z_{0i} = \frac{\cos\omega_c T - Q_i^2}{1 + Q_i^2} \pm j \frac{\sqrt{(2Q_i^2 + 1 - \cos\omega_c T)(\cos\omega_c T + 1)}}{1 + Q_i^2}.$$

From the above relation, the absolute value of z_{0i} becomes unity which means that z_{0i} exists on the unit circle.

Next, we compare the real parts of $e^{j\omega_c T}$ and z_{0i} .

$$\operatorname{Re}[e^{j\omega_c T}] - \operatorname{Re}[z_{0i}] = \frac{Q_i^2(1+\cos\omega_c T)}{1+Q_i^2} > 0$$

Therefore, as shown in Fig. 7-2, it can be seen that z_{0i} exists within the frequency region of $|\omega| > \omega_c$. Q.E.D.

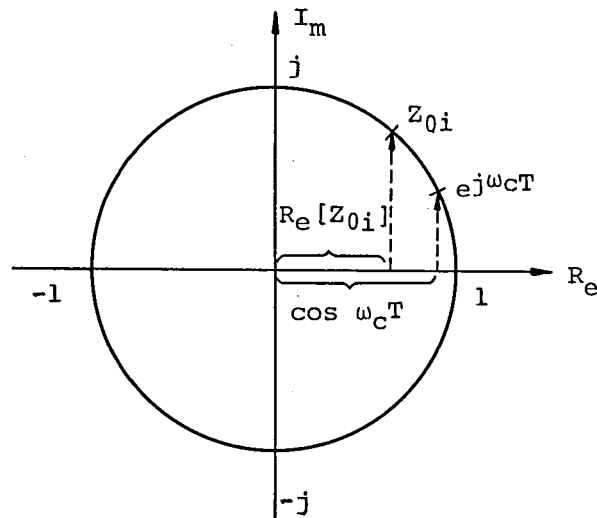


Fig. 7-2 The relation between z_{0i} and ω_c .

The stopband minimum effective loss A_S of a stopband equiripple transfer function can be expressed by⁽⁷⁴⁾

$$A_S = 20 \log_{10} \left| \frac{1}{K} \right| - 20 \log_{10} \left| \frac{1}{H(e^{j0})} \right|$$

where

$$H(e^{j0}) = K \frac{G_e \left[\prod_{i=1}^n (\zeta + \zeta_i) \right]_{\zeta = \sin \frac{\omega_c T}{2}}}{\prod_{i=1}^n \left[\left(\cos \frac{\omega_c T}{2} \right) \cdot \left(\frac{1 - z_i}{1 + z_i} \right) \right]} \quad (7-13)$$

If we choose $|H(e^{j0})| = 1^{(74)}$, then

$$|K| = 10^{-\frac{A_S}{20}} \quad (7-14)$$

Since $A_S > 0$ and, without loss of generality, we can determine $K > 0$, we have $0 < K < 1$.

With the condition of $|H(e^{j0})| = 1$, that is, the condition of 0dB at zero frequency, the time response approximation is carried out.

7-3 ODD DEGREE TRANSFER FUNCTION

An odd degree transfer function satisfying $|H(e^{j0})| = 1$ is introduced, in which all the attenuation poles exist on the unit circle, that is, in Eq.(7-10), $\text{Re}[\zeta] > 0$ and $\text{Re}[\zeta_i] > 0$ for all i 's are satisfied. From the stable condition, all the zeros $z_i (i = 1, 2, \dots, n)$ in the denominator must exist inside the unit circle. We will obtain z_1 satisfying $|H(e^{j0})| = 1$ for a given z_2, z_3, \dots, z_n set in z_i . Eq.(7-13) can be rewritten as

$$H(e^{j0}) = K \frac{\text{SIN} \frac{\omega_c T}{2} \cdot U_0 + \zeta_1 \cdot G_0}{\left(\text{COS} \frac{\omega_c T}{2}\right)^n \cdot \frac{1-z_1}{1+z_1} \cdot \prod_{i=2}^n \left(\frac{1-z_i}{1+z_i}\right)} \quad (7-15)$$

where

$$U(\zeta) = U_n \left[\prod_{i=2}^n (\zeta + \zeta_i) \right]$$

$$G(\zeta) = G_e \left[\prod_{i=2}^n (\zeta + \zeta_i) \right]$$

and

$$U_0 = U(\text{SIN } \frac{\omega_c T}{2})$$

$$G_0 = G(\text{SIN } \frac{\omega_c T}{2}) .$$

Substituting Eq.(7-15) into $|H(e^{j\omega})| = 1$, we have

$$\begin{aligned} & (\text{SIN } \frac{\omega_c T}{2} \cdot U_0 + \zeta_1 \cdot G_0)^2 \\ &= \left(\frac{1}{K} (\text{COS } \frac{\omega_c T}{2})^n \cdot \frac{1-\zeta_1}{1+\zeta_1} \cdot \prod_{i=2}^n \left(\frac{1-\zeta_i}{1+\zeta_i} \right) \right)^2 . \end{aligned} \quad (7-16)$$

From Eq.(7-10), we have

$$\left(\frac{1-\zeta_1}{1+\zeta_1} \right)^2 = \frac{2\zeta_1^2 - (1-\text{COS}\omega_c T)}{1 + \text{COS}\omega_c T} \quad (7-17)$$

Putting

$$L = \prod_{i=2}^n \left(\frac{1-\zeta_i}{1+\zeta_i} \right)$$

and combining Eq.(7-16) and Eq.(7-17), we can obtain a second order equation on ζ_1 .

$$\begin{aligned}
 & (G_0^2 - \frac{2}{1+\cos\omega_{CT}} \cdot ((\cos \frac{\omega_{CT}}{2})^n \cdot \frac{L}{K})^2) \zeta_1^2 \\
 & + 2G_0 \cdot \sin \frac{\omega_{CT}}{2} \cdot U_0 \cdot \zeta_1 \\
 & + \sin^2 \frac{\omega_{CT}}{2} \cdot U_0^2 + \frac{1-\cos\omega_{CT}}{1+\cos\omega_{CT}} \cdot ((\cos \frac{\omega_{CT}}{2})^n \cdot \frac{L}{K})^2 = 0 .
 \end{aligned}
 \tag{7-18}$$

On Eq.(7-18), we have a following Lemma.

[Lemma 7-1]

If $|K| < 1$, there exists a real ζ_1 satisfying Eq.(7-18).

(Proof) In Eq.(7-18), its discriminative equation D becomes

$$\begin{aligned}
 D = & \frac{4}{K^2} \cdot \frac{((\cos \frac{\omega_{CT}}{2})^n \cdot L \cdot \sin \frac{\omega_{CT}}{2})^2}{(1 + \cos\omega_{CT})^2} \\
 & \cdot (((\cos \frac{\omega_{CT}}{2})^n \cdot \frac{L}{K})^2 - \cos^2 \frac{\omega_{CT}}{2} (G_0^2 - U_0^2)) .
 \end{aligned}$$

Using the relation

$$G_0^2 - U_0^2 = (\cos \frac{\omega_{CT}}{2})^{2(n-1)} \cdot L^2 , \tag{7-19}$$

D can be written as follows :

$$D = \frac{4}{K^2} \cdot \frac{((\cos \frac{\omega_c T}{2})^n \cdot L \cdot \sin \frac{\omega_c T}{2})^2}{(1 + \cos \omega_c T)^2} \\ \cdot (\cos \frac{\omega_c T}{2})^{2n} \cdot L^2 \cdot (\frac{1}{K^2} - 1)$$

Since $0 < K < 1$, D is positive which means that Eq.(7-18) has two real roots. Q.E.D.

Since both sides in Eq.(7-17) are real, z_1 is also real. z_1 is obtained by Eq.(7-17) as follows:

$$z_1 = \frac{-(\zeta_1^2 + \cos \omega_c T) \pm \sqrt{4 \cos^2 \frac{\omega_c T}{2} (\zeta_1^2 - \sin^2 \frac{\omega_c T}{2})}}{\zeta_1^2 - 1} \quad (7-20)$$

The product of above two roots becomes unity and, hence, there exists a real z_1 inside the unit circle.

In Eq.(7-9), $\text{Re}[\zeta_i]$ ($i = 1, 2, \dots, n$) must be all positive in order that all the attenuation poles exist on the unit circle. Here, we have a following theorem.

[Theorem 7-2]

In n th odd degree stopband equiripple transfer function $H(z^{-1})$, for a given $n-1$ z_2, z_3, \dots, z_n set, put $H'(z^{-1})$ as

$$H'(z^{-1}) = K \frac{G_e \left[\prod_{i=2}^n (\zeta + \zeta_i) \right]}{\prod_{i=2}^n \left[(2 \cos \frac{\omega_c T}{2}) \left(\frac{1 - z_i z^{-1}}{(1 + z^{-1})(1 + z_i)} \right) \right]}. \quad (7-21)$$

If $0 < K < 1$ and $H'(e^{j0}) < 1$, there exists a positive real ζ_1 satisfying $H(e^{j0}) = 1$. ζ_1 can be obtained by solving Eq.(7-18) as follows:

$$\zeta_1 = \frac{-G_0 \cdot \sin \frac{\omega_c T}{2} \cdot U_0 - \sqrt{D}}{G_0^2 - \left(\left(\cos \frac{\omega_c T}{2} \right)^{n-1} \cdot \frac{L}{K} \right)^2}. \quad (7-22)$$

(Proof) In Eq.(7-18), the first order and the constant coefficients are both positive. If the second order coefficient is positive, Eq.(7-18) has two negative real roots, and if negative, Eq.(7-18) has one negative real root and one positive real root. In order that ζ_1 is positive, the following relation must hold.

$$G_0^2 - \left(\left(\cos \frac{\omega_c T}{2} \right)^{n-1} \cdot \frac{L}{K} \right)^2 < 0.$$

The above relation can be rewritten as follows:

$$K \frac{G_0}{\left(\cos \frac{\omega_c T}{2} \right)^{n-1} \cdot L} = K \frac{G_e \left[\prod_{i=2}^n (\zeta + \zeta_i) \right] \zeta = \sin \frac{\omega_c T}{2}}{\prod_{i=2}^n \left[\left(\cos \frac{\omega_c T}{2} \right) \left(\frac{1 - z_i}{1 + z_i} \right) \right]} = H'(e^{j0}) < 1.$$

Since, in Eq.(7-15), $\text{Re}[\zeta_i] > 0$, $|z_i| < 1$ ($i = 1, 2, \dots, n$), $\text{SIN}(\omega_c T/2) > 0$, $\text{COS}(\omega_c T/2) > 0$ and $K > 0$, the numerator and denominator in Eq.(7-15) are both positive and hence, we have $H(e^{j0}) = 1$ for $|H(e^{j0})| = 1$. Q.E.D.

z_1 can be obtained by taking a solution that its absolute value is smaller than unity in the two solutions in Eq.(7-20). Theorem 7-2 says that the condition $H'(e^{j0}) < 1$ is needed in order that there exists a positive real ζ_1 satisfying $|H(e^{j0})| = 1$ for a given positive $\text{Re}[\zeta_i]$ ($i = 2, 3, \dots, n$) set in Eq.(7-9).

7-4 EVEN DEGREE TRANSFER FUNCTION

An even degree transfer function satisfying $|H(e^{j0})| = 1$ is introduced, in which all the attenuation poles exist on the unit circle. The denominator zeros z_i are divided into two parts of z_1, z_2 and z_3, z_4, \dots, z_n . It should be noted here that, if z_1 is complex, z_2 is complex conjugate, and if z_1 is real, z_2 is also real. Eq.(7-13) can be rewritten as follows:

$$H(e^{j0}) = K \frac{(\text{SIN}^2 \frac{\omega_c T}{2} + \zeta_1 \zeta_2) G_0 + (\zeta_1 + \zeta_2) \text{SIN} \frac{\omega_c T}{2} \cdot U_0}{(\text{COS} \frac{\omega_c T}{2})^n \cdot \frac{1-z_1}{1+z_1} \cdot \frac{1-z_2}{1+z_2} \cdot \prod_{i=3}^n \left(\frac{1-z_i}{1+z_i} \right)}$$

(7-23)

where

$$U(\zeta) = U_n \left[\prod_{i=3}^n (\zeta + \zeta_i) \right]$$

$$G(\zeta) = G_e \left[\prod_{i=3}^n (\zeta + \zeta_i) \right]$$

$$U_0 = U(\text{SIN } \frac{\omega_c T}{2})$$

$$G_0 = G(\text{SIN } \frac{\omega_c T}{2})$$

Substituting Eq.(7-23) in $|H(e^{j0})| = 1$, we have

$$\begin{aligned} & ((\text{SIN}^2 \frac{\omega_c T}{2} + \zeta_1 \zeta_2) G_0 + (\zeta_1 + \zeta_2) \text{SIN} \frac{\omega_c T}{2} \cdot U_0)^2 \\ &= (\frac{1}{K} (\text{COS} \frac{\omega_c T}{2})^n \cdot \frac{1-z_1}{1+z_1} \cdot \frac{1-z_2}{1+z_2} \cdot \prod_{i=3}^n \frac{1-z_i}{1+z_i})^2 \end{aligned} \quad (7-24)$$

From Eq.(7-10), similarly in Eq.(7-17), we have

$$(\frac{1-z_1}{1+z_1})^2 \cdot (\frac{1-z_2}{1+z_2})^2 = (\frac{2\zeta_1^2 + \text{COS} \omega_c T - 1}{1 + \text{COS} \omega_c T}) \cdot (\frac{2\zeta_2^2 + \text{COS} \omega_c T - 1}{1 + \text{COS} \omega_c T})$$

In Eq.(7-24), putting

$$L = \prod_{i=3}^n \frac{1-z_i}{1+z_i} \quad \text{and} \quad \frac{1}{K'} = \frac{1}{K} (\text{COS} \frac{\omega_c T}{2})^n,$$

we have a second order equation on $(\zeta_1 + \zeta_2)$.

$$\begin{aligned}
& \sin^2 \frac{\omega_c T}{2} \left[U_0^2 + 4 \frac{L^2}{K'^2} \cdot \frac{1}{(1 + \cos \omega_c T)^2} \right] \cdot (\zeta_1 + \zeta_2)^2 \\
& + 2G_0 U_0 \cdot \left(\sin^2 \frac{\omega_c T}{2} + \zeta_1 \zeta_2 \right) \cdot \sin \frac{\omega_c T}{2} \cdot (\zeta_1 + \zeta_2) \\
& + \left(\sin^2 \frac{\omega_c T}{2} + \zeta_1 \zeta_2 \right)^2 \cdot \left[G_0^2 - 4 \cdot \frac{L^2}{K'^2} \cdot \frac{1}{(1 + \cos \omega_c T)^2} \right] = 0.
\end{aligned}
\tag{7-25}$$

Eq.(7-25) is an equation to obtain $(\zeta_1 + \zeta_2)$ of the stopband equiripple transfer function satisfying $|H(e^{j0})| = 1$ for a given positive real $\zeta_1 \zeta_2$. On Eq.(7-25), we have a following Lemma and Theorem.

[Lemma 7-2]

If $|K| < 1$ for a given positive real $\zeta_1 \zeta_2$, there exists a real $(\zeta_1 + \zeta_2)$ satisfying Eq.(7-25).

[Theorem 7-3]

In nth even degree stopband equiripple transfer function $H(z^{-1})$, for given $n-2$ z_3, z_4, \dots, z_n set and a positive real $\zeta_1 \zeta_2$, put $H'(z^{-1})$ as

$$H'(z^{-1}) = K \frac{G_e \left[\prod_{i=3}^n (\zeta + \zeta_i) \right]}{\prod_{i=3}^n \left[\left(2 \cos \frac{\omega_c T}{2} \right) \left(\frac{1 - z_i z^{-1}}{(1 + z^{-1})(1 + z_i)} \right) \right]}. \tag{7-26}$$

If $0 < K < 1$ and $H'(e^{j0}) < 1$, there exists a positive real $(\zeta_1 + \zeta_2)$ satisfying $H(e^{j0}) = 1$. $(\zeta_1 + \zeta_2)$ can be obtained by solving

Eq. (7-25) as follows:

$$\zeta_1 + \zeta_2 = \frac{\text{SIN}^2 \frac{\omega_c T}{2} + \zeta_1 \zeta_2}{\text{SIN} \frac{\omega_c T}{2}} \cdot \frac{-G_0 U_0 + \left(\left(\text{COS} \frac{\omega_c T}{2} \right)^{n-2} \cdot \frac{L}{K} \right)^2 \cdot \sqrt{1-k^2}}{U_0^2 + \left(\left(\text{COS} \frac{\omega_c T}{2} \right)^{n-2} \cdot \frac{L}{K} \right)^2} \quad (7-27)$$

(Proof) In Eq. (7-25), the first order and the second order coefficients are both positive. If the constant coefficient is positive, Eq. (7-25) has two negative real roots and if negative, Eq. (7-25) has one negative real root and one positive real root. In order to have a positive real $(\zeta_1 + \zeta_2)$, the following relation must hold.

$$G_0^2 - \frac{4L^2}{K'^2} \cdot \frac{1}{(1 + \text{COS} \omega_c T)^2} < 0$$

It leads to a relation

$$K \frac{G_0}{\left(\text{COS} \frac{\omega_c T}{2} \right)^{n-2} \cdot L} = K \frac{G e^{\left[\prod_{i=3}^n (\zeta + \zeta_i) \right]} \zeta = \text{SIN} \frac{\omega_c T}{2}}{\prod_{i=3}^n \left[\left(\text{COS} \frac{\omega_c T}{2} \right) \left(\frac{1 - z_i}{1 + z_i} \right) \right]} = H'(e^{j\omega}) < 1$$

Since the numerator and the denominator in Eq. (7-23) are both positive, we have $H(e^{j\omega}) = 1$ for $|H(e^{j\omega})| = 1$.

Q.E.D.

Through the similar discussion for an odd degree case, z_1 and z_2 can be obtained by Eq. (7-20). Theorem 7-3 says that the condition $H'(e^{j\omega}) < 1$ is needed in order that there exist

positive $\text{Re}[\zeta_1]$ and $\text{Re}[\zeta_2]$ satisfying $|H(e^{j0})| = 1$ for a given positive $\text{Re}[\zeta_i]$ ($i = 3, 4, \dots, n$) set in Eq.(7-9) and $\zeta_1\zeta_2$.

7-5 THE CASE NOT SATISFYING $H'(e^{j0}) < 1$

If we take some ζ_i as $\text{Re}[\zeta_i] < 0$, some of the attenuation poles do not exist on the unit circle. In this case, the frequency characteristics show a slowly rising attenuation loss curve. However, some cases have better time response.

7-5-1 Odd Degree Case

In the case not satisfying the condition $H'(e^{j0}) < 1$ shown in Section 7-3, the second order coefficient in Eq. (7-18) is positive. Consequently, as shown in the proof of Theorem 7-2, Eq.(7-18) has two negative real roots on ζ_1 . It means that, even if we take $\text{Re}[\zeta_i] > 0$ for all i excluding ζ_1 , one of the pairs of zeros in the numerator of Eq.(7-9) is not on the unit circle. Here we have a following theorem.

[Theorem 7-4]

In n th odd degree stopband equiripple transfer function, if $H'(e^{j0}) \geq 1$ for $H'(z^{-1})$ in Eq.(7-21), there exists a negative real ζ_1 satisfying $H(e^{j0}) = -1$.

(Proof) Since Lemma 7-1 holds, ζ_1 is real. In the case of $H'(e^{j0}) = 1$, ζ_1 can be easily obtained by Eq.(7-18).

$$\zeta_1 = \frac{-\sin \frac{\omega_c T}{2} \cdot (U_0^2 + ((\cos \frac{\omega_c T}{2})^{n-1} \cdot \frac{L}{K})^2)}{2G_0 U_0}$$

We see that ζ_1 is negative. Substituting the above ζ_1 into the numerator in Eq.(7-15), we have

$$\sin \frac{\omega_c T}{2} \cdot U_0 + \zeta_1 \cdot G_0 = \sin \frac{\omega_c T}{2} \cdot \frac{(U_0^2 - G_0^2)}{2U_0}$$

Since, from Eq.(7-19), the numerator in Eq.(7-15) is negative and the denominator is positive, we have $H(e^{j0}) = -1$. In the case of $H'(e^{j0}) > 1$, Eq.(7-18) has two negative real roots which are given by

$$\zeta_1 = \frac{-G_0 \cdot \sin \frac{\omega_c T}{2} \cdot U_0 \pm \sqrt{D}}{G_0^2 - ((\cos \frac{\omega_c T}{2})^{n-1} \cdot \frac{L}{K})^2} \quad (7-28)$$

Here the following two cases exist.

(1) The case that we take a positive sign for composite sign in Eq.(7-28).

Substituting Eq.(7-28) into the numerator in Eq.(7-15),

$$\begin{aligned} & \sin \frac{\omega_c T}{2} \cdot U_0 + \zeta_1 \cdot G_0 \\ &= \frac{-\sin \frac{\omega_c T}{2} \cdot \frac{L^2}{K^2} (\cos \frac{\omega_c T}{2})^{2(n-1)} \cdot (U_0 \frac{1}{K} - G_0 \sqrt{\frac{1}{K^2} - 1})}{G_0^2 - ((\cos \frac{\omega_c T}{2})^{n-1} \cdot \frac{L}{K})^2} \end{aligned} \quad (7-29)$$

Since $H'(e^{j0}) > 1$, the denominator in this equation is positive. We investigate the sign of the parenthesis in the numerator in Eq.(7-29).

$$\begin{aligned} \frac{U_0^2}{K^2} - G_0^2 \left(\frac{1}{K^2} - 1 \right) &= G_0^2 - \frac{1}{K^2} (G_0^2 - U_0^2) \\ &= G_0^2 - \left(\cos \frac{\omega_c T}{2} \right)^{n-1} \cdot \left(\frac{L}{K} \right)^2 > 0 \end{aligned}$$

Hence, we have

$$\sin \frac{\omega_c T}{2} \cdot U_0 + \zeta_1 \cdot G_0 < 0$$

Since the denominator in Eq.(7-15) is positive, we have $H(e^{j0}) = -1$.

(2) The case that we take a negative sign for composite sign in Eq. (7-28).

In this case, since the parenthesis in the numerator in Eq. (7-29) is changed by

$$\left(U_0 \frac{1}{K} + G_0 \sqrt{\frac{1}{K^2} - 1} \right),$$

similarly, we have $H(e^{j0}) = -1$.

Q.E.D.

If we permit a negative ζ_1 like the above case, there exists a negative real ζ_1 satisfying $H(e^{j0}) = 1$ in the case of $H'(e^{j0}) < 1$.

Consequently, for an odd degree transfer function, we can have four cases satisfying $|H(e^{j0})| = 1$, including the result of Section 7-3, which are shown in Table 7-1.

Table 7-1 : Four cases satisfying $|H(e^{j0})| = 1$ (n: odd)

No.	ζ_1	$H'(e^{j0})$	$H(e^{j0})$	Composite sign of Eq. (7-28)
1	Positive	< 1	$= 1$	-
2	Negative	< 1	$= 1$	+
3	Negative	≥ 1	$= -1$	-
4	Negative	≥ 1	$= -1$	+

7-5-2 Even Degree Case

In the case not satisfying the condition $H'(e^{j0}) < 1$ shown in Section 7-4, the constant coefficient in Eq. (7-25) is positive. We have a following theorem.

[Theorem 7-5]

In nth even degree stopband equiripple transfer function $H(z^{-1})$, if $H'(e^{j0}) \geq 1$ for $H'(z^{-1})$ in Eq. (7-26), there exists a negative real $(\zeta_1 + \zeta_2)$ satisfying $H(e^{j0}) = -1$. In this case we can have $(\zeta_1 + \zeta_2)$ as

$$\zeta_1 + \zeta_2 = \frac{\sin^2 \frac{\omega_c T}{2} + \zeta_1 \zeta_2}{\sin \frac{\omega_c T}{2}} \cdot \frac{-G_0 U_0 \pm ((\cos \frac{\omega_c T}{2})^{n-2} \cdot \frac{L}{K})^2 \cdot \sqrt{1-K^2}}{U_0^2 + ((\cos \frac{\omega_c T}{2})^{n-2} \cdot \frac{L}{K})^2} \quad (7-30)$$

Similarly, in the case of $H'(e^{j0}) < 1$, there exists a negative real $(\zeta_1 + \zeta_2)$ satisfying $H(e^{j0}) = 1$. We can have four cases satisfying $|H(e^{j0})| = 1$, including the result of Section 7-4, which are shown in Table 7-2.

Table 7-2 : Four cases satisfying $|H(e^{j0})| = 1$ (n: even)

No.	$\zeta_1 + \zeta_2$	$H'(e^{j0})$	$H(e^{j0})$	Composite sign of Eq.(7-30)
1	Positive	< 1	$= 1$	+
2	Negative	< 1	$= 1$	-
3	Negative	≥ 1	$= -1$	+
4	Negative	≥ 1	$= -1$	-

7-6 TIME RESPONSE APPROXIMATION

Through the former discussions, we can see that in an odd degree case, for a given z_2, z_3, \dots, z_n set, the numerator in Eq.(7-9) and z_1 are determined so that the stopband equiripple transfer function with the condition $|H(e^{j0})| = 1$ can be obtained. In an even degree case, for a given z_3, z_4, \dots, z_n and $\zeta_1 \zeta_2$ sets, the numerator in Eq.(7-9) and $(\zeta_1 + \zeta_2)$ are determined. Therefore, in the time response approximation, we have a z_2, z_3, \dots, z_n parameter set for an odd degree case and z_3, z_4, \dots, z_n and $\zeta_1 \zeta_2$ parameter sets for an even degree case.

In addition, since we can permit the time difference m_0T (m_0 : integer) and the amplitude difference k between the ideal time response and an output time response, we can have m_0 and k with z_i as the parameters. Furthermore, for obtaining better approximation, we may add the all-pass networks which become the parameters, too. (73)

An iterative Chebyshev approximation method by the reference (32) is used here.

The ideal time response for M time sample points is given by $g(mT)$ ($m = 1, 2, \dots, M$). Multiplying an output time response $h(mT + m_0T)$ by k , taking the difference between the two responses and then, multiplying it by the weighting factor W_m , we minimize the maximum error.

$$E_{\max} = \max_{m=1, 2, \dots, M} (W_m(k \cdot h(mT + m_0T) - g(mT))).$$

Putting the transfer function $H(z^{-1})$ as

$$H(z^{-1}) = \frac{\sum_{r=1}^R b_r z^{-r}}{(1 + \sum_{s=1}^S a_s z^{-s})},$$

its impulse response $h(mT)$ can be calculated by the following relation. (82)

$$h(mT) = b_m - a_1 \cdot h((m-1)T) - a_2 \cdot h((m-2)T) \dots - a_m \cdot h(0).$$

(7-31)

By considering the digital filter realization, we can permit $-H(z^{-1})$ other than $H(z^{-1})$. Therefore, when starting an approximation, for a given initial parameters set, each impulse response is computed for four cases of $H(z^{-1})$ and $-H(z^{-1})$ in No. 1 and No. 2 in Tables 7-1 and 7-2 in the case of $H'(e^{j\omega}) < 1$, and then we may start the iterative approximation for a transfer function showing better approximation to the ideal time response. In the case of $H'(e^{j\omega}) \geq 1$, there exist four cases of $H(z^{-1})$ and $-H(z^{-1})$ in No. 3 and No. 4 in the above Tables. We may choose one of them.

7-7 FLOW CHART OF COMPUTER PROGRAM

The flow chart of computer program is shown in Fig. 7-3.

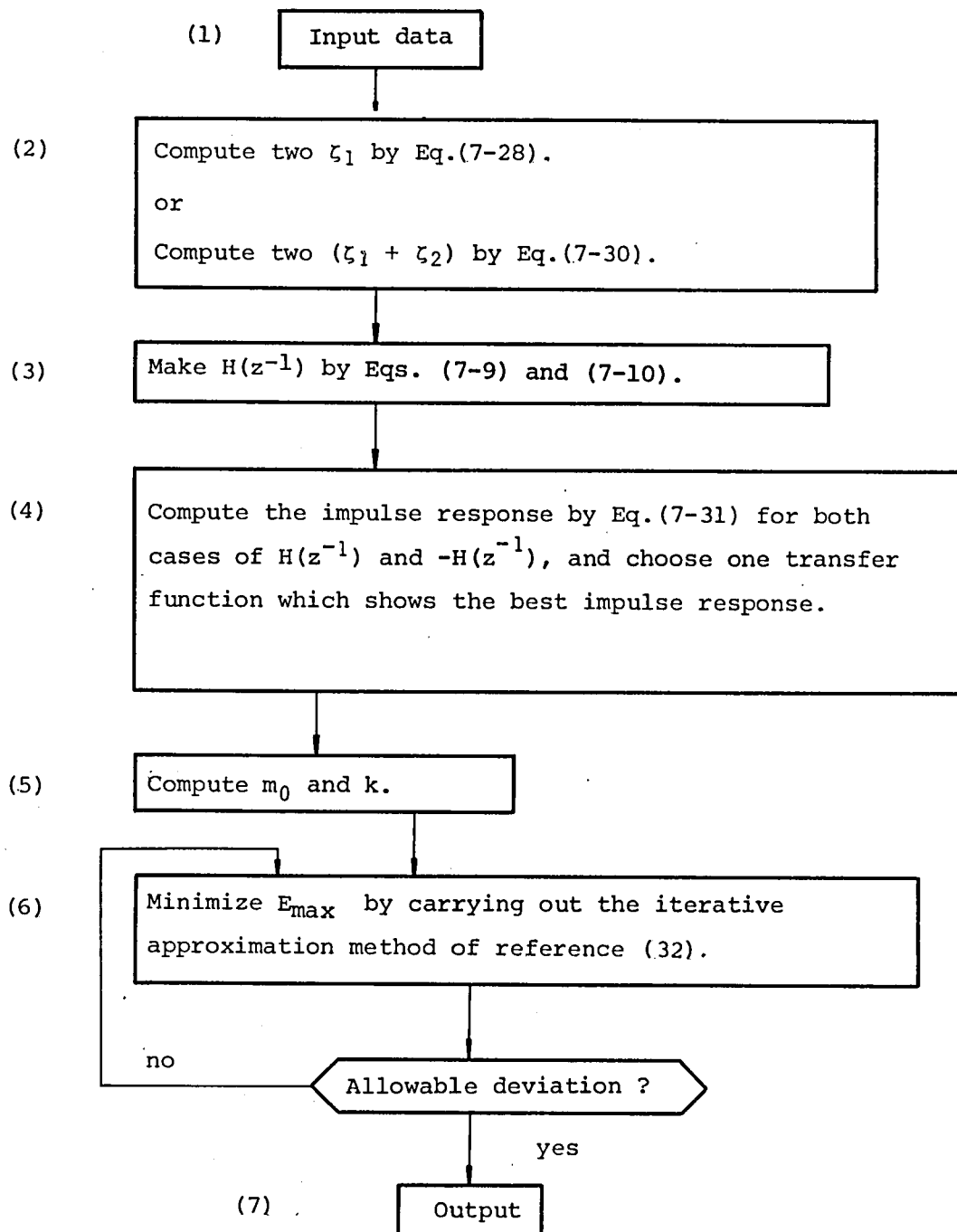


Fig. 7-3 Flow chart of computer program.

According to the flow chart, we will explain it briefly.

(1) Input data are stopband edge frequency: ω_c , stopband attenuation A_s , degree : n , initial parameters : z_1 (z_2, z_3, \dots, z_n for odd degree or z_3, z_4, \dots, z_n and $\zeta_1 \zeta_2$ for even degree), allpass network transfer function, ideal time response : $g(mT)$, weighting : W_m , and allowable deviation.

(2) For odd degree, ζ_1 is computed by Eq.(7-28) which has two solutions equivalent to No. 1, 2 or No. 3, 4 in Table 7-1. For even degree, $(\zeta_1 + \zeta_2)$ is computed by Eq.(7-30) which has two solutions equivalent to No. 1, 2 or No. 3, 4 in Table 7-2.

(3) The two transfer functions $H(z^{-1})$ with equiripple stopband attenuation satisfying $|H(e^{j0})| = 1$ are computed by Eq. (7-9) and (7-10).

(4) The impulse responses for the two transfer functions $H(z^{-1})$ are computed together with the case of $-H(z^{-1})$ by Eq. (7-31) and, then, one transfer function which shows the best time response, is chosen.

(5) The time difference m_0T and the amplitude difference k between the ideal time response and the output time response are simultaneously computed.

(6) By carrying out the iterative approximation, E_{\max} is minimized. This process is repeated until E_{\max} becomes smaller than the allowable deviation.

(7) Output data are transfer function, frequency characteristics, time response, and E_{\max} .

7-8 DESIGN EXAMPLES

Our method will be applied to the design of Nyquist roll off IIR digital filter. The ratio of the Nyquist rate T_N and the sampling rate T is taken to be an integer. It is required that an impulse response has unity at one sample point and zero at other Nyquist sample points. $g(mT)$ for the ideal time response is given at these Nyquist sample points. We can obtain k and m_0 by computing the peak value of time response and time which gives that peak value.

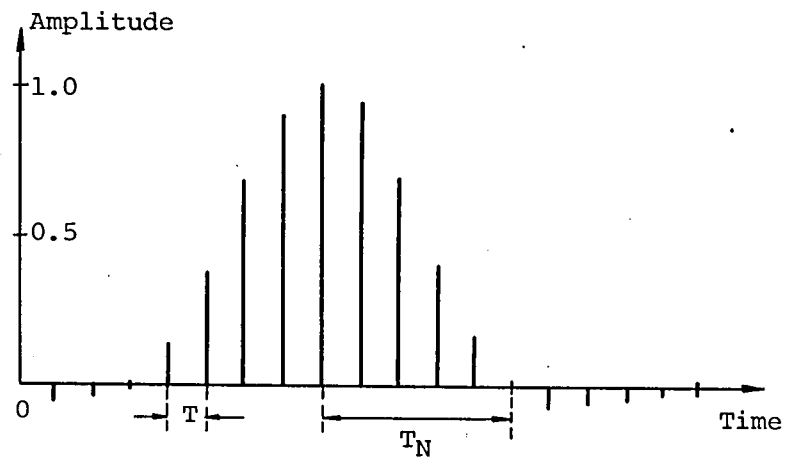
(Example 1) Nyquist frequency: 1kHz (T_N : 0.5 msec)
Sampling frequency: 10kHz (T : 0.1 msec)
Stopband edge frequency: 1.8kHz (80% roll off)
Minimum effective stopband loss: 40dB

We use a fifth degree IIR filter and a second degree all-pass network.

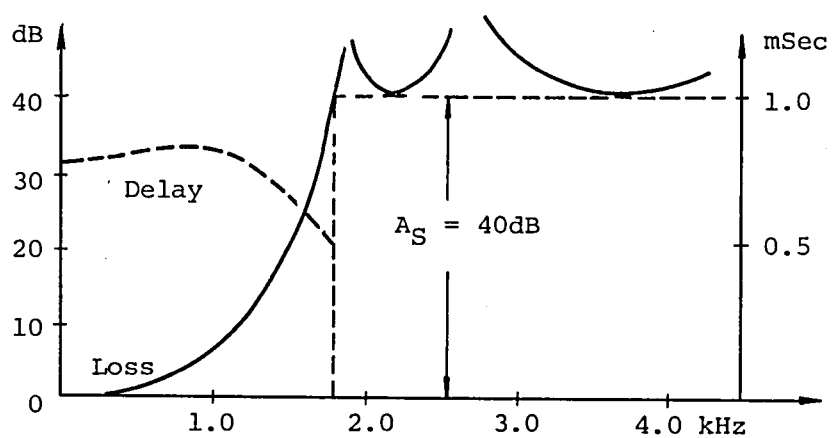
Since IIR filter with No. 1 in Table 7-1 is used here, two pairs of attenuation poles exist both on the unit circle. The transfer function is shown in Table 7-3. An impulse response and frequency characteristics are shown in Fig. 7-4. Maximum error was 8.5×10^{-4} .

Table 7-3

ZEROS	POLES	
0.37394±j0.92745	0.54927±j0.24940	IIR filter
-0.16149±j0.98687	0.48619±j0.47143	
-1.0	-0.04265	
3.07354±j1.9487x10 ⁻³	0.32535±j2.0628x10 ⁻⁴	All-pass network



(a) Time response



(b) Frequency response

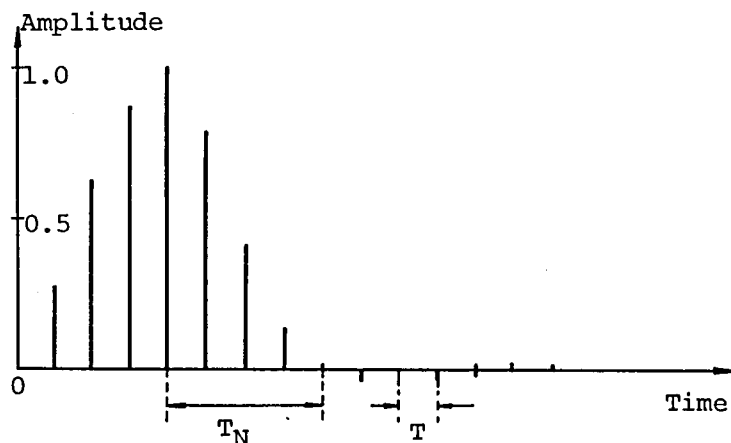
Fig. 7-4 Example 1 of Nyquist roll off digital filter.

(Example 2) Nyquist frequency: 1kHz (T_N : 0.5 msec.)
 Sampling frequency: 8kHz (T : 0.125 msec.)
 Stopband edge frequency: 1.8kHz (80% roll off)
 Minimum effective stopband loss: 30dB

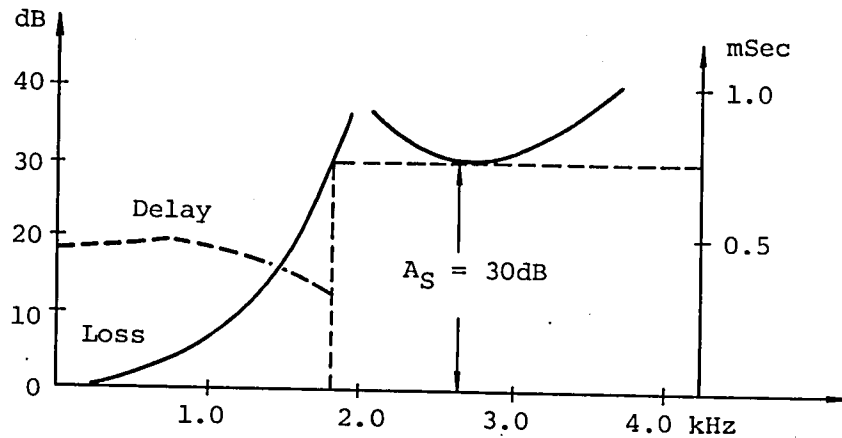
Since a fifth degree IIR filter with No. 3 in Table 7-1 and $-H(z^{-1})$ is used, one pair of attenuation pole does not exist on the unit circle. The transfer function is shown in Table 7-4. An impulse response and frequency characteristics are shown in Fig. 7-5. Maximum error was 1.2×10^{-4} .

Table 7-4

ZEROS	POLES
$0.01981 \pm j0.9998$	$0.31378 \pm j0.38417$
-0.47590×10^{-3}	$0.38200 \pm j0.18215$
-0.21012×10^4	0.0019957
-1.0	



(a) Time response



(b) Frequency response

Fig. 7-5 Example 2 of Nyquist roll off digital filter.

7-9 CONCLUSION

Introducing the transfer function of IIR low pass filter with equiripple stopband attenuation in a sampled system, a time response approximation method holding the minimum effective stopband attenuation in a constant was described. The results obtained in this chapter are listed below.

- (1) In order to hold the minimum effective stopband attenuation in a constant, the condition $|H(e^{j\omega})| = 1$ is required.
- (2) The condition in order that all the attenuation poles exist on the unit circle is given by $H'(e^{j\omega}) < 1$.

(3) In the case of $H'(e^{j0}) \geq 1$, if some of the attenuation poles exist off the unit circle, the condition $|H(e^{j0})| = 1$ can be satisfied.

(4) There are eight cases satisfying $|H(e^{j0})| = 1$ for $H(z^{-1})$ and $-H(z^{-1})$, odd degree and even degree. Starting the iterative approximation, one of them is chosen as an initial parameter set most fitted to the desired time response.

(5) The design examples of Nyquist roll off digital filter with excellent performances are shown.

Theoretically, the simultaneous approximation method in the time domain and frequency domain of IIR digital filter, have been developed. This work have practically contributed to the development of data transmission system with excellent performances in low cost and minimum hardware.

CHAPTER 8. CONCLUDING REMARKS

In this paper, the computer aided design methods on several types of electrical filters used in electrical communication systems, were described.

The results obtained throughout the course of this study are summarized as follows.

- (1) A new approximation theory for a filter with an arbitrary weighted Chebyshev transfer function both in the passband and in the stopband have been developed, in which a reasonable cut off frequency and an optimum degree are automatically determined.
- (2) A practical realization theory together with a synthesis procedure have been found for a reactance band stop filter with unsymmetric transfer characteristics.
- (3) The CAD method carrying out automatically the Norton transformation process in the course of band pass ladder filter design have been developed.
- (4) The design theory have been introduced for a four-port directional filter without compensation networks, used in a submarine cable system.
- (5) The digital filter design program system DINETS have been developed, in which three phase programs in filter design: approximation, synthesis and analysis programs,

are included.

(6) The time response approximation method has been developed for IIR low pass filter with specified equiripple stopband attenuation, used in a data transmission system.

This study has developed the computer aided filter design theories for both the continuous and sampled systems, and the results have made fruitful contributions for the filter design techniques and for the reduction of the necessary hardware, which lead to reduction in total cost of the system.

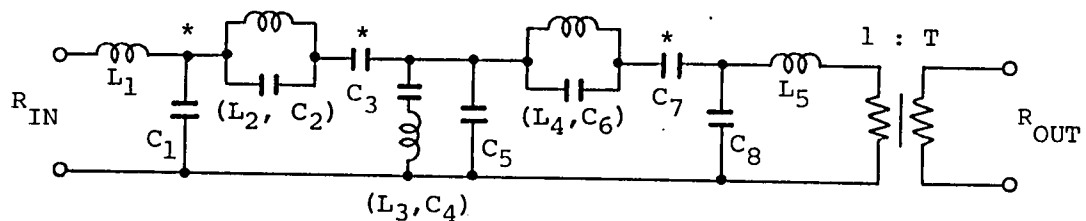
Furthermore, the design automation by CAD method made other contributions which were to release the filter engineers from their time consuming works.

The filter will be the key component in LSI era and play a more and more important role in future in electrical communication systems. By this reason, further efforts must be devoted to advance the filter design theory fitted to the new filter technology.

APPENDIX NORTON TRANSFORMATION PROCESS

Example 1

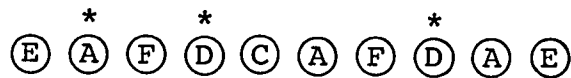
(Step 1) Original circuit



$$1:\phi_1 \quad 1:\phi_2 \quad 1:\phi_3 \quad \phi_3 = \frac{1}{T\phi_1\phi_2} \sqrt{\frac{R_{out}}{R_{in}}}$$

* means the position of ideal transformers.

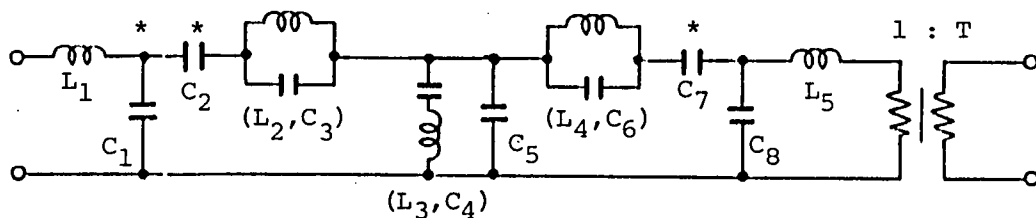
Topology expression



C_1 , C_3 and C_7 in step 1 are specified.

(Step 2)

Since C_1 in step 1 can not make Norton transformation, the order of (L_2, C_2) and C_3 in step 1 is changed.



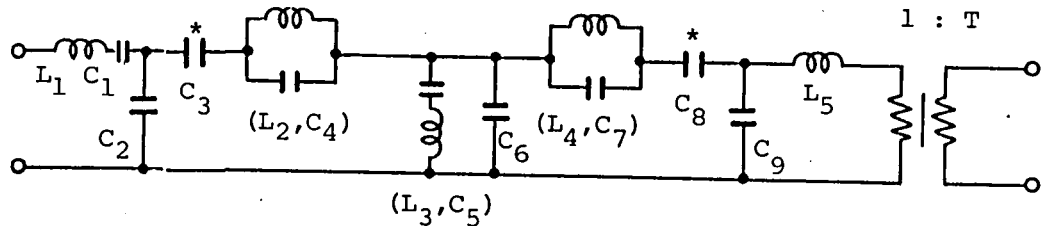
Topology expression

$$\textcircled{E} \textcircled{A}^* \textcircled{D}^* \textcircled{F} \textcircled{C} \textcircled{A} \textcircled{F} \textcircled{D}^* \textcircled{A} \textcircled{E} \quad \phi_1$$

C_1 , C_2 and C_7 in step 2 are specified.

(Step 3)

Since C_1 in step 2 can make Norton transformation, T type transformation is carried out by C_1 and C_2 in step 2.



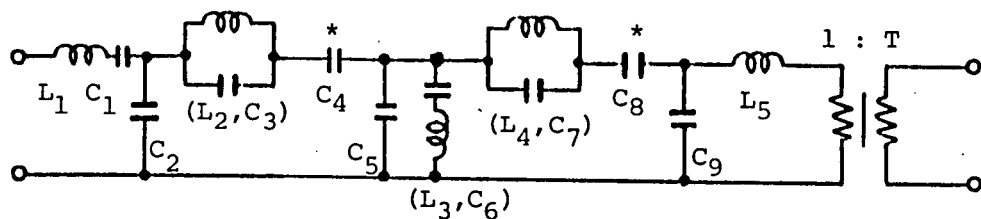
Topology expression

$$\textcircled{E} \textcircled{D} \textcircled{A} \textcircled{D}^* \textcircled{F} \textcircled{C} \textcircled{A} \textcircled{F} \textcircled{D}^* \textcircled{A} \textcircled{E}$$

C_3 and C_8 in step 3 are specified.

(Step 4)

Since C_3 in step 3 can not make Norton transformation, the order of C_3 and (L_2, C_4) in step 3, and the order of (L_3, C_5) and C_6 in step 3 are changed.



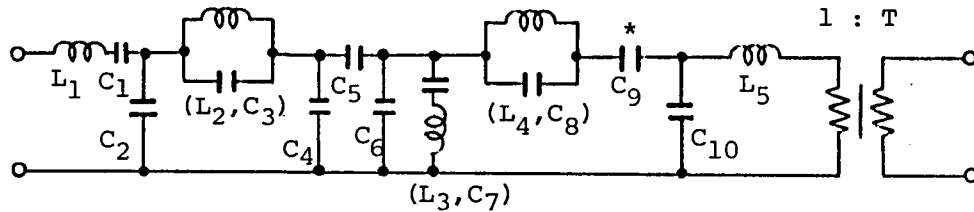
Topology expression

$$\begin{array}{ccccccccccc} \textcircled{\text{E}} & \textcircled{\text{D}} & \textcircled{\text{A}} & \textcircled{\text{F}} & \textcircled{\text{D}}^* & \textcircled{\text{A}} & \textcircled{\text{C}} & \textcircled{\text{F}} & \textcircled{\text{D}}^* & \textcircled{\text{A}} & \textcircled{\text{E}} \\ & & & & \hline & & & & & & & & & & \phi_2 \end{array}$$

C_4 and C_8 in step 4 are specified.

(Step 5)

Since C_4 in step 4 can make Norton transformation, Π type transformation is carried out by C_4 and C_5 in step 4.



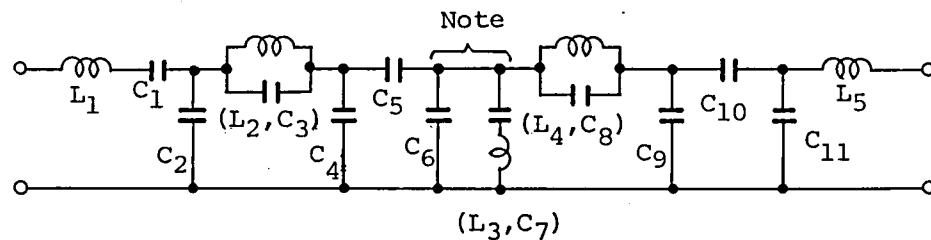
Topology expression

$$\begin{array}{ccccccccccc} \textcircled{\text{E}} & \textcircled{\text{D}} & \textcircled{\text{A}} & \textcircled{\text{F}} & \textcircled{\text{A}} & \textcircled{\text{D}} & \textcircled{\text{A}} & \textcircled{\text{C}} & \textcircled{\text{F}} & \textcircled{\text{D}}^* & \textcircled{\text{A}} & \textcircled{\text{E}} \\ & & & & & & & & & \hline & & & & & & & & & & \phi_3 \end{array}$$

C_9 in step 5 is specified.

(Step 6) Final circuit

Since C_9 in step 5 can make Norton transformation, Π type transformation is carried out by C_9 and C_{10} in step 5.



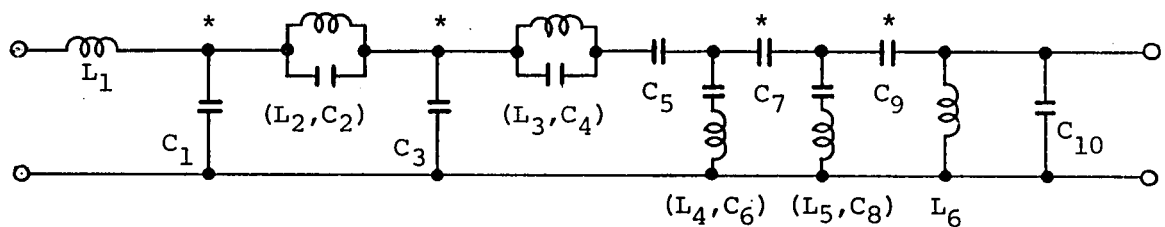
Topology expression

(E) (D) (A) (F) (A) (D) (A) (C) (F) (A) (D) (A) (E)

Note: This arm (C_6 and (L_3, C_7)) will be carried out 2-terminal impedance transformation.

Example 2

(Step 1) Original circuit



* means the position of ideal transformers.

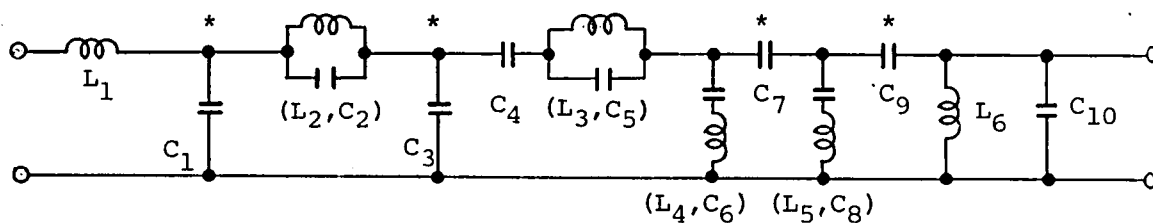
Topology expression

(E) (A) (F) (A) (F) (D) (C) (D) (C) (D) (B) (A)

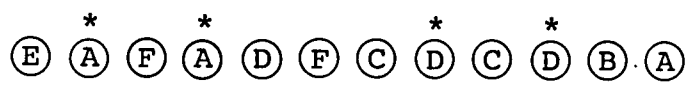
C_1 , C_3 , C_7 and C_9 in step 1 are specified.

(Step 2)

Though C_1 in step 1 is specified, C_1 can not make Norton transformation. So, we investigate C_3 in step 1. Though C_3 in step 1 can not make Norton transformation too, the order of (L_3, C_4) and C_5 in step 1 can be changed.



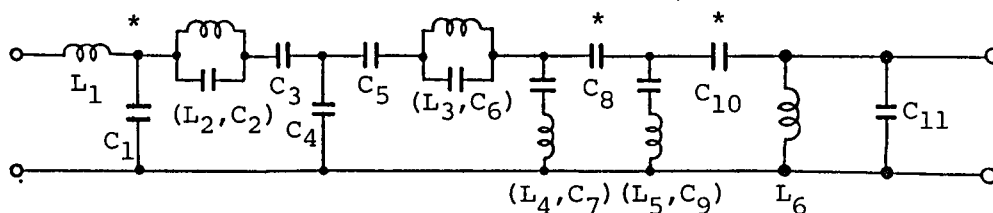
Topology expression



C_1 , C_3 , C_7 and C_9 in step 2 are specified.

(Step 3)

Since C_3 in step 2 can make Norton transformation, T type transformation is carried out by C_3 and C_4 in step 2.



Topology expression

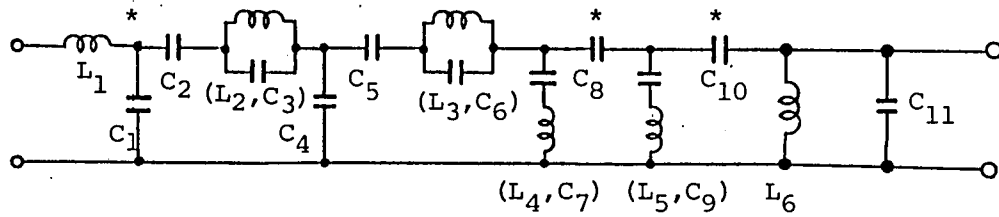


C_1 , C_8 and C_{10} in step 3 are specified.

(Step 4)

Here, we again investigate C_1 in step 3. Though C_1 in

step 3 can not still make Norton transformation, the order of (L_2, C_2) and C_3 in step 3 can be changed.



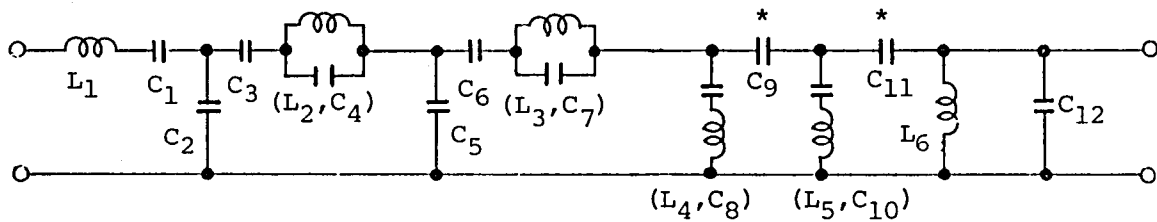
Topology expression



C_1 , C_8 and C_{10} in step 4 are specified.

(Step 5)

Since C_1 in step 4 can make Norton transformation, T type transformation is carried out by C_1 and C_2 in step 4.



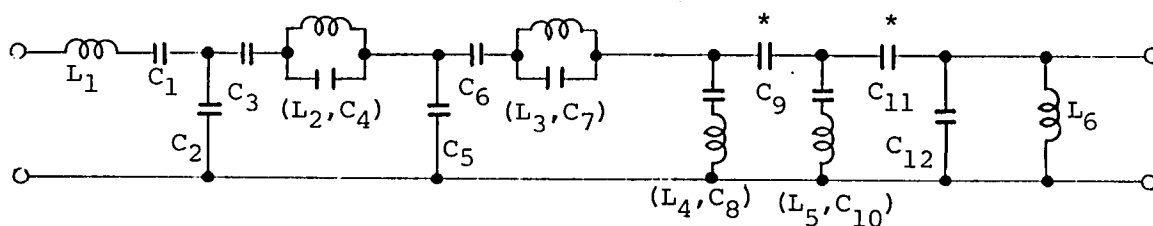
Topology expression



C_9 and C_{11} in step 5 are specified.

(Step 6)

Though C_9 in step 5 is specified, C_9 can not make Norton transformation. So, we investigate C_{11} in step 5. Though C_{11} in step 5 can not make Norton transformation too, the order of L_6 and C_{12} in step 5 can be changed.



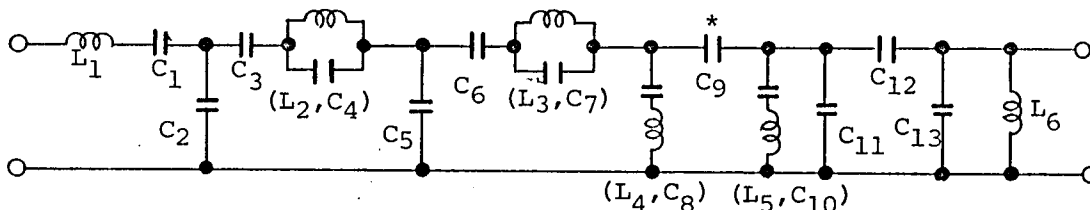
Topology expression

(E) (D) (A) (D) (F) (A) (D) (F) (C) (D) (C) (D) (A) (B)

C_9 and C_{11} in step 6 are specified.

(Step 7)

Since C_{11} in step 6 can make Norton transformation, Π type transformation is carried out by C_{11} and C_{12} in step 6.



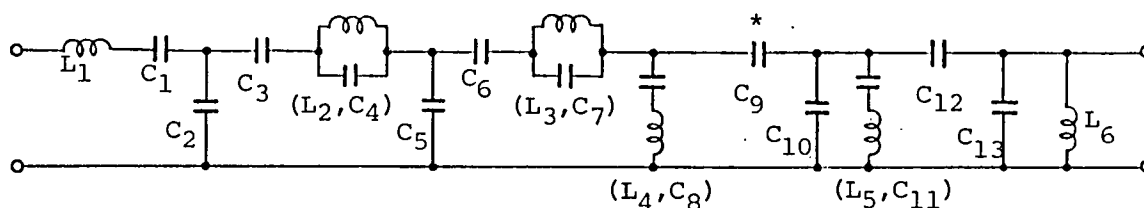
Topology expression

(E) (D) (A) (D) (F) (A) (D) (F) (C) (D) (C) (A) (D) (A) (B)

C_9 in step 7 is specified.

(Step 8)

Here, we again investigate C_9 in step 7. Though C_9 in step 7 can not still make Norton transformation, the order of (L_5, C_{10}) and C_{11} in step 7 can be changed.



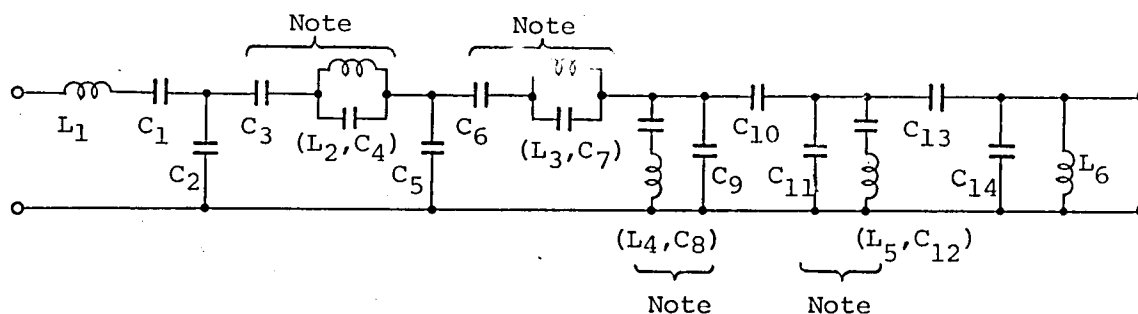
Topology expression



C_9 in step 8 is specified.

(Step 9) Final circuit

Since C_9 in step 8 can make Norton transformation, Π type transformation is carried out by C_9 and C_{10} in step 8.



Topology expression

Ⓔ Ⓓ Ⓐ Ⓓ Ⓕ Ⓐ Ⓓ Ⓕ Ⓒ Ⓐ Ⓓ Ⓐ Ⓒ Ⓓ Ⓐ Ⓑ

Note: These arms (C_3 and (L_2, C_4) , C_6 and (L_3, C_7) ,
 (L_4, C_8) and C_9, C_{11} and (L_5, C_{12})) will be
carried out 2-terminal impedance transformation.

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